Logic Programming

Backtracking and Cut

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Contents

Finitely Many Alternatives

Simplest way: Several facts match against the question.

Example

```
father(mary, george).
father(john, george).
father(sue, harry).
father(george, edward).

?- father(X, Y).
X=mary, Y=george ;
X=john, Y=george ;
X=sue, Y=harry ;
X=george, Y=edward
```

The answers are generated in the order in which the facts are given.

Repeating the Same Answer

Old answers do not influence newer ones: same answer can be returned several times.

Example

```
father(mary, george).
father(john, george).
father(sue, harry).
father(george, edward).

?- father(_,X).
X = george ;
X = george ;
X = harry ;
X = edward
```

`george` returned twice because George is the father of both Mary and John.

Embedding Does Not Matter

Backtracking happens in the same way if the alternatives are embedded more deeply.

Example

```
father(mary, george).
father(john, george).
father(sue, harry).
father(george, edward).

child(X,Y) :- father(Y,X)

?- child(X,Y).
X = george, Y = mary ;
X = george, Y = john ;
X = harry,  Y = sue ;
X = edward, Y = george
```

Mixing facts and Rules

If facts and rules are mixed, the alternatives follow again in the order in which things are presented.

Example

```
person(adam).
person(X) :- mother(X, Y).
person(eve).
mother(cain, eve).
mother(abel, eve).
mother(jabal, adah).
mother(tubalcain, zillah).
```

```
?- person(X).
X = adam ;
X = cain ;
X = abel ;
X = jabal ;
X = tubalcain ;
X = eve
```

Multiple Goals with Multiple Solutions

More interesting case: two goals, each with several solutions.

Example

```
pair(X, Y) :-
    boy(X),
    girl(Y).

boy(johm).
boy(marmaduke).
boy(bertram).
boy(charles).

girl(griselda).
girl(ermitrude).
girl(brunhilda).
```

```
?- pair(X, Y).
X = john, Y = griselda ;
X = john, Y = ermintrude ;
X = john, Y = brunhilde ;
X = marmaduke, Y = griselda ;
X = marmaduke, Y = ermintrude ;
X = marmaduke, Y = brunhilde ;
X = bertram, Y = griselda ;
...
```

12 solutions.

Infinite Number of Possibilities

Sometimes we want to generate an infinite number of possibilities.

It might not be known in advance how many of them needed.

Example

```
is_integer(0).
is_integer(X) :-
    is_integer(Y),
    X is Y+1.

?- is_integer(X).
X = 0 ;
X = 1 ;
X = 2 ;
...
```

How does it work?

Member and Multiple Solutions

Most rules give rise to alternative solutions if they are used for goals that contain many uninstantiated variables.

Example

```
member(X, [X|_]).
member(X, [_|Y]) :-
    member(X, Y).

?- member(A, X).
X = [A|_] ;
X = [_,A|_] ;
X = [_,_,A|_] ;
X = [_,_,_,A|_] ;
...
```

There is a way to tell PROLOG to discard choices: The "cut".

The "Cut"

Cut (written "!") tells the system which previous choices need not to be considered again when it backtracks.
Advantages:

- The program will run faster. No time wasting on attempts to re-satisfy certain goals.
- The program will occupy less memory. Less backtracking points to be remembered.

Example of Cut

Reference library:

- Determine which facilities are available.
- If one has an overdue book can only use the *basic facilities*.
- Otherwise can use the *general facilities*.

Reference Library

Example

```
facility(Person, Facility) :-
    book_overdue(Person, Book),
    !,
    basic_facility(Facility).
facility(Person, Facility) :-
    general_facility(Facility).

basic_facility(reference).
basic_facility(enquiries).

additional_facility(borrowing).
additional_facility(inter_library_loan).

general_facility(X) :- basic_facility(X).
general_facility(X) :- additional_facility(X).
```

Reference Library

Example

```
book_overdue('C. Watzer', book10089).
book_overdue('A. Jones', book29907).
...
client('C. Watzer').
client('A. Jones').
...
?- client(X), facility(X,Y).
```

How does it proceed?

Reference Library

The effect of cut:

- If a client has an overdue book, then only allow her/him the basic facilities.
- Don’t bother going through all the clients overdue books.
- Don’t remember any other rule about facilities.

The Effect of Cut

In general, when a cut is encountered as a goal

- The system becomes committed to all choices made since the parent goal was invoked.
- All other alternatives are discarded.
- An attempt to re-satisfy any goal between the parent goal and the cut goal will fail.

Common Uses of Cut

Three main cases:

1. To tell the system that it found the right rule for a particular goal. *Confirming the choice of a rule*.
2. To tell the system to fail a particular goal without trying for alternative solutions. *Cut-fail combination.*
3. To tell the system to terminate the generation of alternative solutions by backtracking. *Terminate a "generate-and-test".*

Confirming the Choice of a Rule

Typical situation:

- We wish to associate several clauses with the same predicate.
- One clause is appropriate if the arguments are of one form, another is appropriate if the arguments have another form.
- Often (but not always) these alternatives can be made disjoint by providing just the argument patterns (e.g., empty list in one clause, and a nonempty list in another.)
- If we cannot specify an exhaustive set of patterns, we may give rules for some specific argument types and gave a "catchall" rule at the end for everything else.

Confirming the Choice of a Rule

Example of the case when an exhaustive set of patterns can not be specified:

Example

```
sum_to(1, 1).
sum_to(N, Res):-
    N1 is N-1,
    sum_to(N1, Res1),
    Res is Res1+N.

?- sum_to(5, X).
 X=15 ;
```

It loops.

Confirming the Choice of a Rule

What happened?

- `sum_to(1,1)` and `sum_to(N,Res)` are not disjoint alternatives.
- `sum_to(1,1)` matches both `sum_to(1,1)` and `sum_to(N,Res)`.
- But if a goal matches `sum_to(1,1)`, there is no reason why it should try the second alternative, `sum_to(N,Res)`.
- Cut the second alternative.

Confirming the Choice of a Rule

Example

```
sum_to(1, 1) :-
    !.
sum_to(N, Res) :-
    N1 is N-1,
    sum_to(N1, Res1),
    Res is Res1+N.

?- sum_to(5, X).
X = 15 ;
false
```

More Usual Situation

- In the previous example we could specify a pattern for the boundary case `sum_to(1,1)`.
- Usually, it is hard to specify pattern if we want to provide extra conditions that decide on the appropriate rule.
- The previous example still loops on goals `sum_to(N,Res)` where `N =< 1`.
- We can put this condition in the boundary case telling PROLOG to stop for such goals.
- But then the pattern can not be specified.

Cut with Extra Conditions

Example

```
sum_to(N, 1) :-
    N =< 1,
    !.
sum_to(N, Res) :-
    N1 is N-1,
    sum_to(N1, Res1),
    Res is Res1+N.
```

Cut and Not

General principle:

- When cut is used to confirm the choice of a rule, it can be replaced with `not`.
- `not(X)` succeeds when `X`, seen as a PROLOG goal, fails.
- Replacing cut with `not` is often considered a good programming style.
- However, it can make the program less efficient.
- Trade-off between readability and efficiency.

Cut and Not

Example (With Cut)

```
sum_to(1, 1) :- !.
sum_to(N, Res) :-
    N1 is N-1,
    sum_to(N1, Res1),
    Res is Res1+N.
```

Example (With Not)

```
sum_to(1, 1).
sum_to(N, Res) :-
    not(N = 1),
    N1 is N-1,
    sum_to(N1, Res1),
    Res is Res1+N.
```

Cut and Not

Example (With Cut)

```
sum_to(1, 1) :-
    N =< 1, !.
    sum_to(N, Res) :-
    N1 is N-1,
sum_to(N1, Res1),
    Res is Res1+N.
```

Example (With Not)

```
sum_to(1, 1) :-
    N =< 1,
sum_to(N, Res) :-
    not(N =< 1), % N > 1
    N1 is N-1,
    sum_to(N1, Res1),
    Res is Res1+N.
```

Double Work

`not` might force PROLOG to try the same goal twice:

```
A :- B, C.
A :- not(B), D.
```

`B` may be tried twice after backtracking.

The "Cut-fail" Combination

`fail.`

- Built-in predicate.
- No arguments.
- Always fails as a goal and causes backtracking.

The "Cut-fail" Combination

`fail` after `cut`:

- The normal backtracking behavior will be altered by the effect of cut.
- Quite useful combination in practice.

The Average Taxpayer

Write a program to determine an average taxpayer.

Two cases:

- Foreigners are not average taxpayers.
- If a person is not a foreigner, apply the general criterion (whatever it is) to find out whether he or she is an average taxpayer.

The Average Taxpayer

Example

```
average_taxpayer(X) :-
    foreigner(X),
    !, fail.
average_taxpayer(X) :-
    satisfies_general_criterion(X).
```

What would happen had we omitted the cut?

The Average Taxpayer

Wrong version, without cut:

Example (Wrong)

```
average_taxpayer(X) :-
    foreigner(X),
    fail.
average_taxpayer(X) :-
    satisfies_general_criterion(X).
```

If there is a foreigner `widslewip` who satisfies the general criterion, the program will incorrectly answer yes on the goal

```
?- average_taxpayer(widslewip).
```

The Average Taxpayer

We can use cut-fail combination to define `satisfies_general_criterion`.

Two cases:

- A person whose spouse earns more than a certain amount (e.g. Euro 3000) does not satisfy the criterion of being an average taxpayer.
- If this is not the case, then a person satisfies the criterion if his income is within a certain interval (e.g. more that Euro 2000 and less than Euro 3000).

The Average Taxpayer

Clauses for `satisfies_general_criterion`.

Example

```
satisfies_general_criterion(X) :-
    spouse(X, Y),
    gross_income(Y, Inc),
    Inc > 3000,
    !, fail.
satisfies_general_criterion(X) :-
    gross_income(X,Inc),
    Inc < 3000,
    Inc > 2000.
```

The Average Taxpayer

We can use cut-fail combination to define `gross_income`.

Two cases:

- A person who gets a pension less than certain amount (e.g. Euro 500), is considered to have no gross income.
- Otherwise, person's gross income is determined as the sum of his/her gross salary and investment income.

The Average Taxpayer

Clauses for `gross_income`.

Example

```
gross_income(X, Y) :-
    receives_pension(X, P),
    P < 500,
    !, fail.
gross_income(X, Y) :-
    gross_salary(X, Z),
    investment_income(X, W),
    Y is Z+W.
```

`not` with Cut and Fail

`not` can be defined in terms of cut and fail.

Example

```
not(P) :-
    call(P),
    !, fail.
not(P).
```

Notation: `\+` is used more often for `not`.

Replacing Cut with `not`

- Cut can be replaced with `not` in cut-fail combination.
- Unlike the first use of cut, this replacement does not affect efficiency.
- However, more reorganization of the program is required.

Example

```
average_taxpayer(X) :-
    not(foreigner(X)),
    not(spouse(X,Y), gross_income(Y,Inc), Inc>3000),
    ⋮
```

Terminating a "Generate-and-Test"

"Generate-and-Test":

- One of the simplest AI search techniques.
- Generate: Generate all possible solutions to a problem.
- Test: Test each to see whether they are a solution.
- A possible solution is generated and then tested.
- If the test succeeds a solution is found.
- otherwise, backtrack to next possible solution.

Tic-Tac-Toe

Tic-Tac-Toe game: Get three in a raw, column, or diagonal:

X		O
O	O	
X	X	X

X	X	O
O	X	
O	X	

O		O
X	O	
X	X	O

Representation:

1	2	3
4	5	6
7	8	9

Tic-Tac-Toe

We will show a part of the program to play Tic-Tac-Toe.

Used predicates:

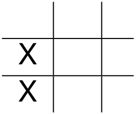
- `var`: built-in predicate. `var(T)` succeeds if `T` is a free variable.
- `arg`: built-in predicate. `arg(N,T,A)` succeeds if `A` is `N`th argument of the term `T`.
- `aline`: defined predicate. Generator of possible lines. For instance, `aline([1,5,9])` is the following line:

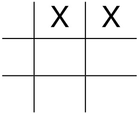
X		
	X	
		X

Part of the Program for Tic-Tac-Toe

The opponent (playing with crosses) is threatening to claim a line:

```
threatening([X,Y,Z], B, X) :-
  empty(X, B), cross(Y, B), cross(Z, B).
```





Part of the Program

Example

```
forced_move(Board, Sq) :-
    aline(Squares),
    threatening(Squares, Board, Sq),
    !.

aline([1,2,3]).
aline([4,5,6]).
aline([7,8,9]).
aline([1,4,7]).
aline([2,5,8]).
aline([3,6,9]).
aline([1,5,9]).
aline([3,5,7]).
```

Part of the Program

Example (Cont.)

```
threatening([X,Y,Z], B, X) :-
    empty(X, B),
    cross(Y, B),
    cross(Z, B).
threatening([X,Y,Z], B, Y) :-
    cross(X, B),
    empty(Y, B),
    cross(Z, B).
threatening([X,Y,Z], B, Z) :-
    cross(X, B),
    cross(Y, B),
    empty(Z, B).
```

`forced_move`

`forced_move` implements "generate-and-test":

- Moves Generated by `alines`: All possible ways that cross can win.
- Moves Tested by `threatening`: If cross can win in the next move.
- If no forced moves are found, then the predicate fails and some other predicate would decide what move to make.

Cut

Suppose embedded in a larger program:

- If `forced_move` successfully finds a move then `Sq` becomes instantiated to the move.
- If, later, a failure occurs (after this instantiation) `forced_move` would retry.
- Cut can prevent PROLOG to search further (which would be futile) and not waste time.
- When we look for forced moves it is only the first solution that is important.

Problems with the Cut

Cut changes behavior of programs:

- Introducing cuts may give a correct behavior when goals are of one form.
- There is no guarantee that anything sensible will happen if goals of another form start appearing.

Problems with the Cut

Example

```
number_of_parents(adam, 0) :- !.
number_of_parents(eve, 0) :- !.
number_of_parents(_, 2).

?- number_of_parents(eve, X).
X = 0 ;
false

?- number_of_parents(john, X).
X = 2 ;
false

?- number_of_parents(eve, 2).
true
```

Problems with the Cut

Example

```
number_of_parents(adam, N) :- !, N=0.
number_of_parents(eve, N) :- !, N=0.
number_of_parents(_, 2).

?- number_of_parents(eve, 2).
false
```

However, it will still not work properly if we give goals such as

```
?- number_of_parents(X, Y).
```