Conference Program

(version of July 29, 2019)

OPSFA 2019

15th International Symposium on

Orthogonal Polynomials, Special Functions and Applications

22 – 26 July 2019

Hagenberg, Austria

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Foreword

Welcome in Hagenberg, Austria, to the 15th International Symposium on Orthogonal Polynomials, Special Functions and Applications! This meeting is organized jointly by the Research Institute for Symbolic Computation (RISC) of the Johannes Kepler University Linz (JKU) and the Johann Radon Institute for Computational and Applied Mathematics (RICAM) of the Austrian Academy of Sciences (ÖAW).

RISC is part of the Department of Mathematics in the Faculty of Engineering and Natural Sciences of JKU and, in addition, it is part of the Softwarepark Hagenberg, an innovative center for research, education, and industrial cooperations founded around 1990 by Bruno Buchberger, who is also the founding chairman of RISC. Since 2009 the director of RISC is Peter Paule. The institute is located in the medieval castle of Hagenberg whose roots go back to the 12th century. Its research area, symbolic computation, is a sub-area of computer mathematics and deals with the algorithmic processing of mathematical symbols that describe for instance numbers, functions, or geometrical objects. Today's advanced mathematics is based heavily on symbolic representations, and therefore, symbolic computation is at the heart of modern mathematics.

RICAM is a research institute of the Austrian Academy of Sciences, which was founded in 2003 by Heinz W. Engl. It is located on the campus of the Johannes Kepler University in Linz, about 20km away from Hagenberg. RICAM focuses on basic research in applied mathematics, and within the institute mathematicians from all around the globe collaborate on common core areas in mathematical modeling, simulation, inverse problems, optimization, symbolic computation, discrete mathematics, and cryptography. The working groups at RICAM provide a broad field of expertise over a whole range of different subjects, and together they create an exciting atmosphere to carry out research in applied mathematics.

This symposium is an event of the SIAM Activity Group on Orthogonal Polynomials and Special Functions. Conferences in the OPSFA series provide a forum for mathematicians, physicists, and computational scientists to communicate recent research results in the areas of orthogonal polynomials and special functions. We hope that this 2019 OPSFA meeting will be as fruitful as the past meetings, and that it inspires you to build on the beautiful mathematics presented here.

We would like to thank our sponsors (see the previous page), whose support was essential in realizing this event. Many thanks to the members of the Scientific Committee (see below), for their advice and help on the scientific aspects of this meeting. Last but not least, we are extremely grateful to our two staff assistants, Tanja Gutenbrunner and Ramona Oehme-Pöchinger, for their hard work in organizing this conference, and to Ralf Hemmecke for his technical assistance.

Christoph Koutschan

(RICAM – Johann Radon Institute for Computational and Applied Mathematics, Austrian Academy of Sciences)

Peter Paule

(RISC – Research Institute for Symbolic Computation, Johannes Kepler University)

Committees

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- Walter van Assche (Katholieke Universiteit Leuven, Belgium)
- Diego Dominici (State University of New York, New Paltz, USA)
- Kathy Driver (University of Cape Town, South Africa)
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- Thomas Trogdon (University of California, Irvine, USA)

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- Peter Paule (RISC, Johannes Kepler University Linz, Austria)

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- Ali Uncu (RISC)
- Elaine Wong (RICAM)

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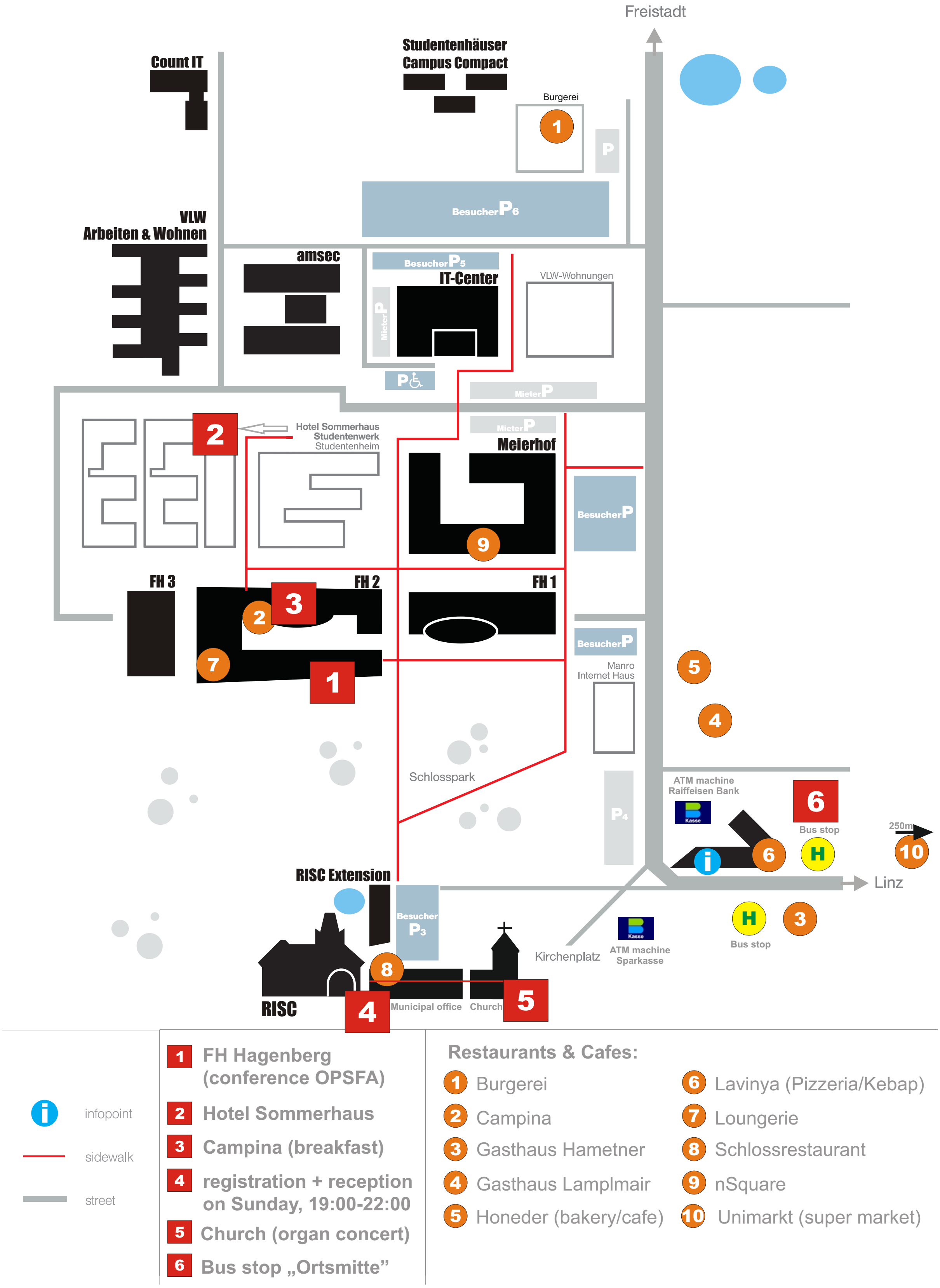
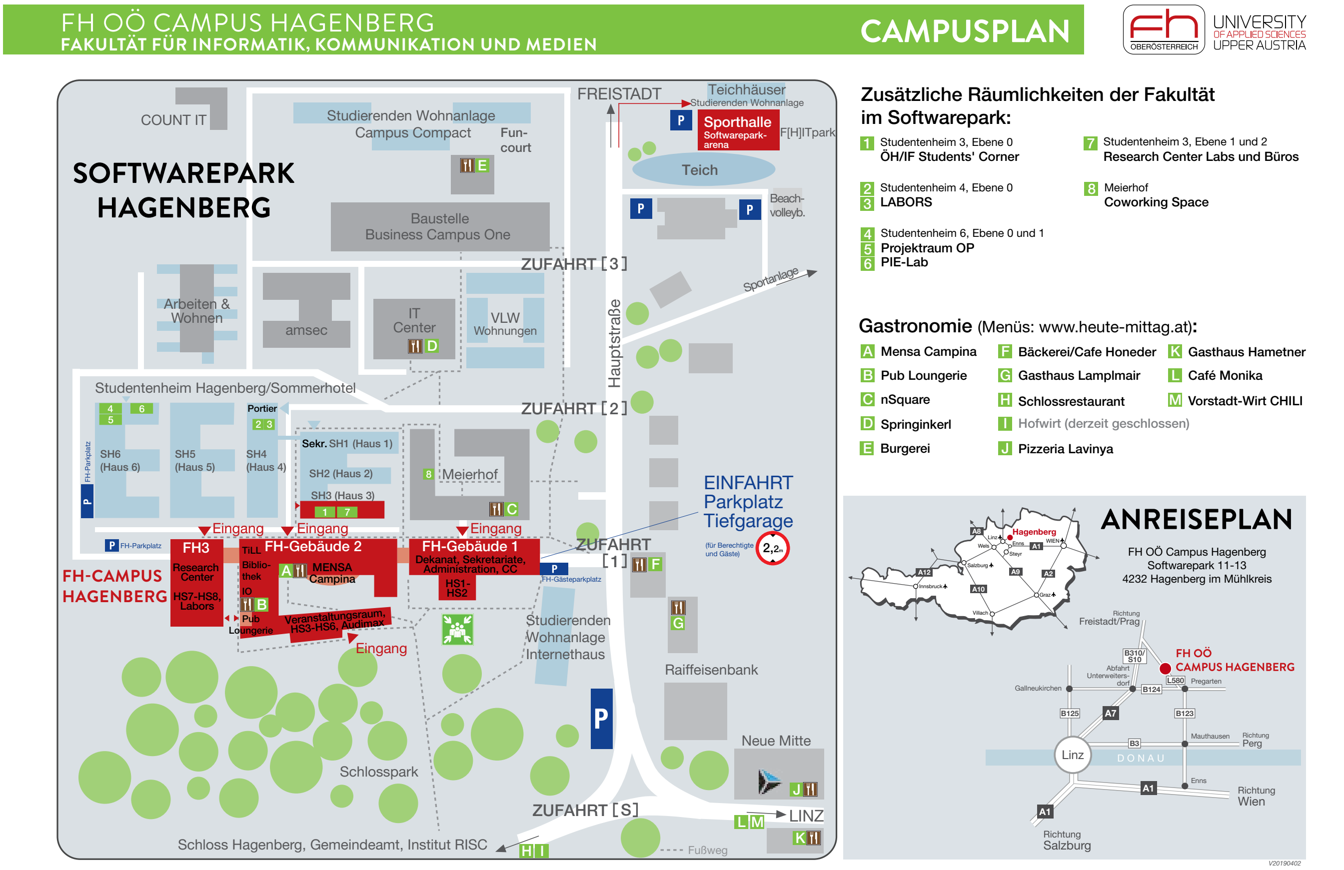
- Tanja Gutenbrunner (RISC)
- Ramona Oehme-Pöchinger (RISC)

Map of Hagenberg Center



Room Plan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7 Studentenheim 3, Ebene 1 und 2 Research Center Labs und Büros
2 3 Studentenheim 4, Ebene 0 LABORS
8 Meierhof Coworking Space
4 5 6 Studentenheim 6, Ebene 0 und 1 Projektraum OP PIE-Lab
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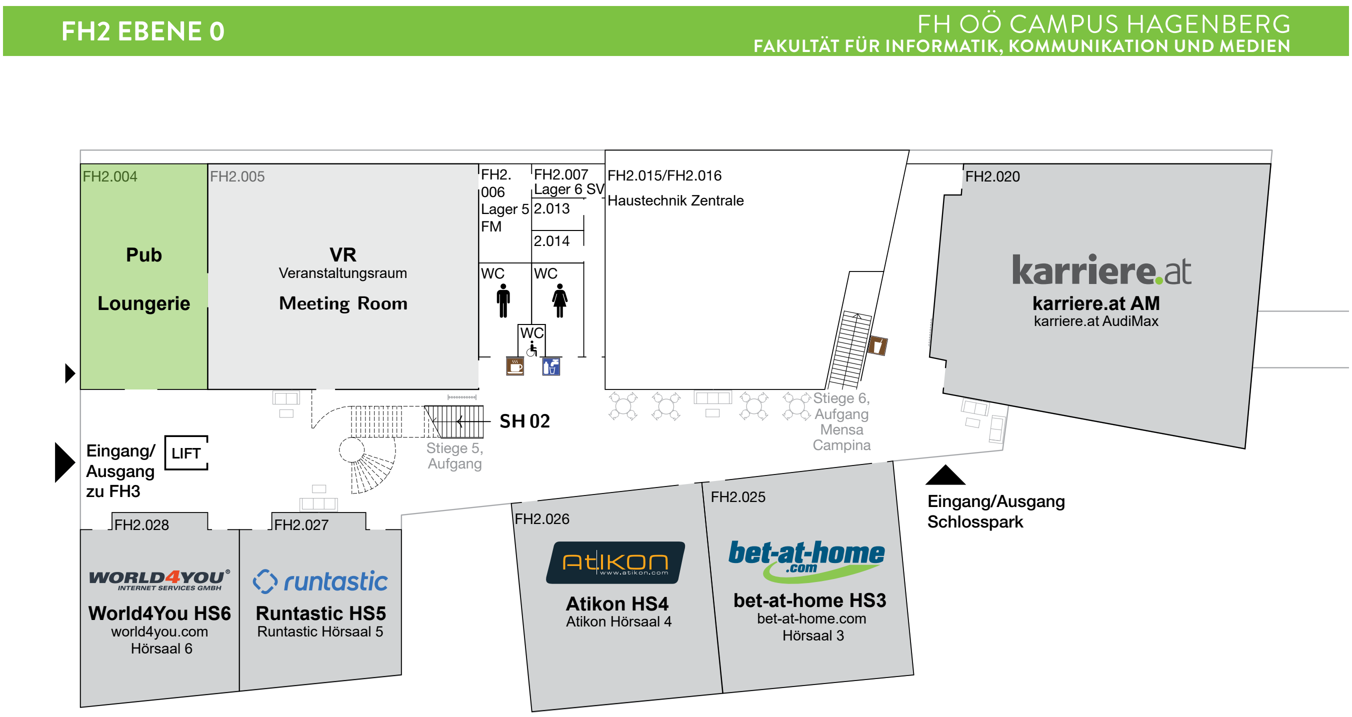
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Conference Program

	Monday	Tuesday	Wednesday	Thursday	Friday
09:00	Krattenthaler	Sokal	Bothner	Vinet	Straub
10:00	Coffee Break	Coffee Break	Coffee Break	Coffee Break	Coffee Break
10:30 – 12:30	MS 02, MS 05, MS 11, MS 07, MS 12	MS 02, MS 05, MS 11, MS 06, MS 03	MS 01, MS 08, MS 09, MS 06, MS 03, MS 12	MS 01, MS 08, MS 09, MS 06, MS 04, MS 10	MS 01, MS 08, MS 12, MS 04
12:30	Lunch	Lunch	Lunch	Lunch	Clarkson (12:00)
14:00	Sodin	Pillwein	Poster Session	Nenciu	Lunch
15:00	Coffee Break	Coffee Break		Coffee Break	
15:30 – 17:30	MS 02, MS 05, MS 11, MS 07, MS 03	MS 01, MS 05, MS 11, MS 07, MS 03	Excursion	MS 01, MS 08, MS 09, MS 10, MS 04	
	Business Meet.	Open Problems	Dinner	Organ Concert	

MS01 Orthogonal polynomials, special functions, and functional equations

MS02 Hypergeometric functions

MS03 Trends on orthogonal polynomials in weighted Sobolev spaces

MS04 Multivariate special functions related to Lie algebras

MS05 Multiple orthogonal polynomials and Hermite-Padé approximation

MS06 Symbolic computation and special functions

MS07 Recent trends in asymptotics

MS08 Asymptotics via non-standard orthogonality

MS09 Extremal polynomials and almost periodicity

MS10 Potential theory and applications to orthogonal polynomials and minimal energy

MS11 Developments in q-series and the theory of partitions

MS12 General session for contributed talks

Mon	Room AM	Room HS 3	Room HS 4	Room HS 5	Room HS 6	Room SH 02
09:00	**Christian Krattenthaler**: Chen Wang's proof of the Borwein conjecture (Chair: Christoph Koutschan)					
10:00	Coffee Break					
10:30	**Olde Daalhuis** Large parameter asymptotics for hypergeometric and Legendre functions	**L.-Lagomasino** Asymptotics of Cauchy biorthogonal polynomials	**Banerjee** New representations for $\sigma(q)$ via reciprocity theorems	**Aniceto** From asymptotics to exact results: unraveling the analytic structure of	**Srinivasan** Integrals of Mellin–Barnes type	
11:00	**Molag** Monodromy of the generalized hypergeometric equation in the Frobe-	**M.-Finkelshtein** Asymptotics of multiple orthogonal polynomials for cubic weight	**Nazaroglu** Modular properties of false theta functions	**Howls** Invisible catastrophes: when to turn an asymptotic blind eye	**Acatrinei** Discrete variations on an old special functions theme	
11:30	**Jankov Maširević** New series representation formulas for modified Bessel function of	**Yattselev** Asymptotics of the recurrence coefficients of multiple orthogo-	**Ciolan** New results on asymptotics and inequalities for partition functions	**Dai** Gaussian unitary ensembles with pole singularities near the soft edge	**Haubold** Symbolic evaluation of *hp*-FEM element matrices on simplices	
12:00	**Cho** Newton diagram for the positivity of ${}_1F_2$ hypergeometric functions and	**Medina Peralta** On matrix Cauchy biorthogonal polynomials	**Schneider** Symbolic Summation, difference ring algorithms and q-applications	**Yao** Distribution of the maximal height of N nonintersecting Bessel paths	**Lalo** Orthogonal polynomials arising from the expansion of first degree poly-	
12:30	Lunch					
14:00	**Mikhail Sodin**: Three tales from one pocket (Chair: Mourad Ismail)					
15:00	Coffee Break					
15:30	**Jiménez Pastor** Symbolic computation for D^n-finite functions	**Chaggara** Some characterization problems related to d-orthogonal polynomial	**Burson** Enumerating simultaneous core partitions into k distinct parts	**Takei** Voros coefficients and the topological recursion for the hypergeometric	**Marcellán,Moreno** New trends on orthogonal polynomials in Sobolev spaces	
16:00	**Blaschke** Hypergeometric form of fundamental theorem of calculus	**Foulquié Moreno** Matrix Laguerre biorthogonal polynomials via the Riemann–Hilbert	**Bhatnagar** On a continued fraction of Ramanujan	**Aoki** The hypergeometric function and WKB solutions	**Behr** Operational methods in the study of Sobolev-Jacobi polynomials	
16:30	**Mansour** A q-Hurwitz zeta function associated with a q-analogue of Bernoulli	**Arvesú Carballo** On some multiple orthogonal polynomials of a discrete variable	**Jennings-Shaffer** Some q-series conjectures related to Rogers-Ramanujan type identi-	**Takahashi** The confluent hypergeometric function and WKB solutions	**Domínguez de la I.** Bispectral Laguerre and Jacobi type polynomials	
17:00	**Das** Functional inequalities for the generalized Wright functions	**Lima** Multiple orthogonal polynomials associated with confluent hypergeo-	**Paule** Modular functions and computer algebra		**Markett** A new symmetric representation of the differential equation for the	
17:30	SIAM-AG OPSF Business Meeting					

Tue	Room AM	Room HS 3	Room HS 4	Room HS 5	Room HS 6	Room SH 02
09:00	**Alan Sokal**: Coefficientwise Hankel-total positivity (Chair: Kathy Driver)					
10:00	Coffee Break					
10:30	**Driver** Sharp parameter range for interlacing of zeros of same degree Laguerre	**Leurs** Laguerre-Angelesco multiple orthogonal polynomials on an r-star	**Kenfack Nangho** Exponential function on nonuniform lattices and solutions to some q-inde-	**Mezzarobba** Rigorous numerical evaluation of D-finite functions in SageMath	**Mañas** Sobolev biorthogonal polynomials and the Gauss-Borel factoriza-	
11:00	**Maier** Hypergeometric transformations based on Hahn and Racah	**Loureiro** Multiple orthogonal polynomials living on a star: ratio asymptotics	**Zafeirakopoulos** Using geometry for computing q-series	**Wei** Implementing finite summation identities of polygamma and	**Szafraniec** "Hermitian + nilpotent" = Sobolev moment problem	
11:30	**Verde-Star** A family of hypergeometric orthogonal polynomial sequences that	**Dyachenko** Discrete multiple orthogonal polynomials on shifted lattices	**Uncu** Multisums related to Rogers–Ramanujan type identities and their	**Jiu** Orthogonal polynomials for higher-order Euler polynomials	**Mínguez** Fourier series of Sobolev polynomials for coherent pairs of Jacobi type	
12:00	**Verschoor** A Bailey type factorization of Horn's H4 hypergeometric function	**Neuschel** On the eigenvalues of Hermitian Brownian motion in critical situations	**Ismail** The Deng-Yang conjecture	**Drmota** Positive systems of polynomial equations	**Pijeira Cabrera** Rational approximation and Sobolev orthogonal polynomials	
12:30	Lunch					
14:00	**Veronika Pillwein**: Orthogonal Polynomials, Special Functions, and Algorithms (Chair: Peter Paule)					
15:00	Coffee Break					
15:30	**Ebisu** Invariants of difference equations and transformation formulae for hy-	**Van Assche** Multiple Askey–Wilson polynomials and related multiple orthogonal po-	**Çetinkaya** Ruscheweyh-type starlike functions of complex order associated with	**Kiro** On functions K and E generated by a sequence of moments	**Mañas-Mañas** Eigenvalues of a differential operator related to classical discrete Sobolev	
16:00	**Iwasaki** Discrete Laplace method and hypergeometric continued fractions	**Schlosser** Multilateral inversion of hypergeometric series	**Smoot** Some new Ramanujan–Kohlberg identities	**Olde Daalhuis** Transition region expansions	**Garza Gaona** On Freud-Sobolev type orthogonal polynomials: asymptotics and zeros	
16:30	**Kimura** Some special functions of matrix integral type and quantum Painlevé equa-	**Cesarano** A survey on biorthogonal polynomials and functions	**Goswami** A q-analogue for Euler's $\zeta(6) = \pi^6/945$	**Nemes** On the Borel summability of WKB solutions near a simple pole	**Molano Molano** On coherence relations between quasi-definite linear functionals and	
17:00	**Haraoka** Asymptotic analysis for a confluent KZ type equation	**Branquinho** Multiple orthogonal polynomials interpretation of some high order Toda	**Radu** Proofs of some q-product identities conjectured by Merca	**Yoo** Hook length property of d-complete posets via q-integrals		
17:30	Open Problems Session (Kathy Driver)					

Wed	Room AM	Room HS 3	Room HS 4	Room HS 5	Room HS 6	Room SH 02
09:00	**Thomas Bothner**: What is . . . a Riemann-Hilbert problem? (Chair: Walter Van Assche)					
10:00	Coffee Break					
10:30	**Cohl** More precise symmetric descriptions for properties of the Askey-Wilson	**Kuijlaars** Non-hermitian orthogonality in random tiling problems	**Kalmykov** Bernstein-type inequalities and geometric function theory	**Wong** Polynomials from $(0, m, s)$-nets and Walsh functions	**Fernández** Sobolev orthogonal polynomials on the triangle	**Jooste** Recurrence equations involving different orthogonal polynomial
11:00	**Ogawara** On algebraic independence of solutions for systems of algebraic	**Deaño** Two applications of non-standard orthogonality	**Zinchenko** Lower bounds for extremal polynomials	**Koutschan** Computer algebra for basic hypergeometric functions	**Figueroa** On Dunkl–Sobolev orthogonal polynomials in the ball involving	**Zhang** Completely monotonic Fredholm determinants
11:30	**Matsubara-Heo** Connection problem of GKZ hypergeometric functions	**Celsus** Construction of the global parametrix for the kissing polynomials	**Dragovic** Periodic ellipsoidal billiards and extremal polynomials	**Schneider** An algorithmic summation theory for indefinite nested sums and prod-	**Marriaga** Coherent pairs of bivariate orthogonal polynomials	**Tyaglov** A generalization of the Silvester-Kac matrix
12:00	**Dzhamay** Discrete Painlevé Equations in Tiling Problems	**Barhoumi** Generalized Jacobi polynomials on a cross	**Schiefermayr** Chebyshev polynomials on circular arcs	**Johansson** The mathematical functions grimoire		**Li** The fractional Green's function by Babenko's approach
12:30	Lunch					
13:30	Poster Session					
14:30	Excursion and Conference Dinner					

Thu	Room AM	Room HS 3	Room HS 4	Room HS 5	Room HS 6	Room SH 02
09:00	**Luc Vinet**: State revival in spin networks, graphs and orthogonal polynomials (Chair: Galina Filipuk)					
10:00	Coffee Break					
10:30	**Yakubovich** Orthogonal polynomials with ultra-exponential weight functions: an	**Miña Díaz** Some results and conjectures on the asymptotic properties of poly-	**Eckhardt** Continued fraction expansions and generalized indefinite strings	**Straub** Interpolated sequences and critical L-values of modular forms	**Xu** Intertwining operator for the dihedral group	**Matos** A new approach for solving equilibrium problems in potential theory
11:00	**Hu** Asymptotics of solutions to the second Painlevé hierarchy	**Groot** Converting planar orthogonality to orthogonality on a contour	**Yuditskii** Martin functions of Fuchsian groups and character automorphic	**Huang** Efficient rational creative telescoping	**De Clercq** The higher rank q-Bannai–Ito algebra and multivariate $(-q)$-Racah	**Berg** A family of entire functions connecting the Bessel function J_1 and
11:30	**Derezinski** On some families of exactly solvable Schrödinger operators	**Wielonsky** Zeros of Faber polynomials for Joukowski airfoils	**Bessonov** Szegő condition and scattering for Dirac operators	**Petkovšek** Inverse Zeilberger's Problem	**Groenevelt** A quantum algebra approach to multivariate Askey–Wilson polynomi-	**Nikolov** New bounds for the extreme zeros of classical orthogonal polynomials
12:00	**Luo** Exceptional extensions of some $q = -1$ classical orthogonal polynomials	**Beckermann** Detecting outliers with Christoffel-Darboux kernels	**Gohlke** Schrödinger operators with substitutive potentials beyond linear	**Imamoglu** Sparse polynomial interpolation with arbitrary orthogonal polyno-	**Ito** Elliptic extension of Gustafson's q-integral of type G_2	**Brauchart** The logarithmic and Riesz minimal energy problem on sets of
12:30	Lunch					
14:00	**Irina Nenciu**: On orthogonal polynomials and the long-time behavior of completely integrable systems (Chair: Paco Marcellán)					
15:00	Coffee Break					
15:30	**Casper** The Matrix Bochner Problem	**López García** Nikishin systems on star-like sets: limiting functions in ratio asymp-	**Eichinger** Periodic coordinates and a magic formula for finite-gap CMV matrices	**Dragnev** Minimal energy problem on the sphere, equilibrium support, and	**Koelink** Convolution identities arising from the Lie superalgebra $\mathfrak{osp}(1\|2)$	
16:00	**Bonneux** Wronskian Appell polynomials and symmetric functions	**Lysov** On the irrationality and the measure of irrationality of	**Lawton** Values distribution of almost periodic functions, spectral factorization	**Criado del Rey** Spherical ensemble with two charges	**Koornwinder** A nonsymmetric version of Okounkov's BC-type interpolation Macdonald	
16:30	**Stevens** Coefficients of Wronskian Hermite polynomials	**Jordaan** Properties of some classes of quasi-orthogonal polynomials	**Garza** CMV block matrices for symmetric matrix measures on the unit circle	**Stylianopoulos** Potential theory on orthogonal polynomials arising from subnor-	**Rösler** Riesz distributions and the Wallach set in Dunkl theory	
17:00	**Costas Santos** Exceptional orthogonal polynomials from the semi-classical point of	**Simanek** Blaschke Products, numerical ranges, and the zeros of orthogonal poly-	**Kononova** Szegő minimum problem and Nevai's conjecture	**Stepaniuk** Hyperuniformity on flat tori	**Rosengren** Multidimensional matrix inversions and elliptic hypergeometric series on	
18:00	Organ Concert in the church of Hagenberg (Christian Krattenthaler)					

Fri	Room AM	Room HS 3	Room HS 4	Room HS 5	Room HS 6	Room SH 02
09:00	**Armin Straub**: Negative thinking and polynomial analogs (Chair: Michael Schlosser)					
10:00	Coffee Break					
10:30	**Pedersen** Nielsen's beta-function and some infinitely divisible distributions	**Świderski** Asymptotics of orthogonal polynomials with unbounded recurrence coef-	**Herscovici** New degenerate Eulerian polynomials	**Filipuk** On the Heun functions	**Seifert** Multivariate Chebyshev polynomials in algebraic signal processing	
11:00	**Tcheutia** Recurrence equations and their classical orthogonal polynomial	**Anbhu** Biorthogonal rational functions involving two parameters and	**Jeffrey** The Gamma function and its inverse		**Shibukawa** Multivariate Meixner, Charlier and Krawtchouk polyno-	
11:30	**Nagoya** On connection problem of q-conformal blocks and its application	**Mbouna** On π_N-coherent pair with index M and order (m, k) of orthogonal	**Brus** Discretization of generalized Chebyshev polynomials of (anti)symmetric		**van de Vijver** Racah problems for the oscillator algebra and $\mathfrak{sl}_n$	
12:00	**Peter A. Clarkson**: Rational solutions of Painlevé equations (Chair: Zeinab Mansour)					
13:00	Lunch					

Chen Wang's proof of the Borwein conjecture

Christian Krattenthaler
(*Universität Wien, Vienna, Austria*)
Time: Monday 22.07., 09:00, Room AM

Abstract: The (so-called) Borwein conjecture arose around 1990 and states that the coefficients in the polynomial

$$(1-q)(1-q^2)(1-q^4)(1-q^5)\cdots(1-q^{3n-2})(1-q^{3n-1})$$

have the sign pattern $+--+--\dots$. This innocent looking prediction has withstood all proof attempts until last year when Chen Wang found a proof that combines asymptotic estimates with a computer verification for "small" n. I shall review the history of the conjecture and the various attempts that have been made to prove it, and then give an overview of Wang's proof. I shall close with further open problems in the same spirit.

Three tales from one pocket

Mikhail Sodin
(*Tel Aviv University, Israel*)
Time: Monday 22.07., 14:00, Room AM

Abstract: I plan to present (and connect) three at first glance unrelated results found in joint works with A. Borichev, A. Kononova, A Nishry and B. Weiss (arXiv:1409.2736, 1701.03407, 1902.00874, 1902.00872)

(1) As all of us know, the exponential decay of the series $\sum_{n\geq 0}(-1)^n x^n/n!$ is a result of incredible cancellations. Quite surprisingly, one can achieve a similar cancellation replacing the sequence $(-1)^n$ by a random complex-valued stationary correlated sequence provided that the origin does not belong to the closed convex hull of the support of the spectral measure of the sequence.

(2) Finitely valued random stationary sequences possess a striking spectral property: if the spectral measure of the sequence has a gap in its support then the sequence must be periodic.

(3) Given a non-negative measure ρ on the unit circle $\mathbb{T}$, the Szegő minimum problem is to find the quantity

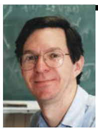
$$e_n(\rho)^2 = \min_{q_1,\dots,q_n} \int_{\mathbb{T}} \left|1 + q_1 t + \ldots + q_n t^n\right|^2 \mathrm{d}\rho(t).$$

A celebrated result, first, proven by Szegő for absolutely continuous measures ρ, and then, independently, by Verblunsky and Kolmogorov in the general case, states that

$$\lim_{n\to\infty} e_n(\rho) = \exp\Big(\frac{1}{2}\int_{\mathbb{T}} \log(\mathrm{d}\rho/\mathrm{d}m)\,\mathrm{d}m\Big),$$

where m is the Lebesgue measure on $\mathbb{T}$ normalized by condition $m(\mathbb{T}) = 1$ and $\mathrm{d}\rho/\mathrm{d}m$ is the Radon-Nikodym derivative. Thus, $\lim_{n\to\infty} e_n(\rho) = 0$ if and only if the measure ρ has a divergent logarithmic integral. In spite of the classical nature of this result, little is known how properties of a measure ρ with divergent logarithmic integral affect the rate of decay of the sequence $e_n(\rho)$. I will present several quantitative results in that direction.

Coefficientwise Hankel-total positivity



Alan Sokal
(*University College London, UK / New York University, USA*)
Time: Tuesday 23.07., 09:00, Room AM

Abstract: A matrix M of real numbers is called *totally positive* if every minor of M is nonnegative. Gantmakher and Krein showed in 1937 that a Hankel matrix $H = (a_{i+j})_{i,j\geq 0}$ of real numbers is totally positive if and only if the underlying sequence $(a_n)_{n\geq 0}$ is a Stieltjes moment sequence. Moreover, this holds if and only if the ordinary generating function $\sum_{n=0}^{\infty} a_n t^n$ can be expanded as a Stieltjes-type continued fraction with nonnegative coefficients:

$$\sum_{n=0}^{\infty} a_n t^n = \cfrac{\alpha_0}{1 - \cfrac{\alpha_1 t}{1 - \cfrac{\alpha_2 t}{1 - \cfrac{\alpha_3 t}{1 - \cdots}}}}$$

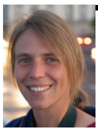
(in the sense of formal power series) with all $\alpha_i \geq 0$. So totally positive Hankel matrices are closely connected with the Stieltjes moment problem and with continued fractions.

Here I will introduce a generalization: a matrix M of polynomials (in some set of indeterminates) will be called *coefficientwise totally positive* if every minor of M is a polynomial with nonnegative coefficients. And a sequence $(a_n)_{n\geq 0}$ of polynomials will be called *coefficientwise Hankel-totally positive* if the Hankel matrix $H = (a_{i+j})_{i,j\geq 0}$ associated to (a_n) is coefficientwise totally positive. It turns out that many sequences of polynomials arising naturally in enumerative combinatorics are (empirically) coefficientwise Hankel-totally positive. In some cases this can be proven using continued fractions, by either combinatorial or algebraic methods; I will sketch how this is done. In many other cases it remains an open problem.

One of the more recent advances in this research is perhaps of independent interest to special-functions workers: we have found branched continued fractions for ratios of contiguous hypergeometric series ${}_rF_s$ for arbitrary r and s, which generalize Gauss' continued fraction for ratios of contiguous ${}_2F_1$. For the cases $s = 0$ we can use these to prove coefficientwise Hankel-total positivity.

Reference: Mathias Pétréolle, Alan D. Sokal and Bao-Xuan Zhu, arXiv:1807.03271

Orthogonal Polynomials, Special Functions, and Algorithms



Veronika Pillwein
(*Johannes Kepler Universität, Linz, Austria*)
Time: Tuesday 23.07., 14:00, Room AM

Abstract: Orthogonal polynomials and special functions are used in many different scientific fields and so are methods from symbolic computation. The study of special functions is part of classical mathematical theory. For instance, people have been deriving linear differential equations or difference equations satisfied by special functions or, starting from a differential equation, determining recurrence relations of coefficient sequences since centuries. Many of the techniques used here are already algorithmic and have been implemented and extended in the recent decades.

A step beyond these classical methods has been initiated by Doron Zeilberger and his holonomic systems approach in the early 1990s. For instance Jacobi polynomials $P_n^{(\alpha,\beta)}(x)$ are a feasible input for these algorithms and linear relations between shifts in the parameters n, α, β and derivatives w.r.t. x can be found and proven automatically.

Some of these algorithms have been generalized to cover also certain non-holonomic input such as Stirling-type sequences. This approach is still closely related to Zeilberger's paradigm. Relying on classical symbolic algorithms such as Gröbner bases computations or Cylindrical Algebraic Decomposition, methods have been developed for finding and proving non-linear algebraic relations and for proving inequalities on special functions satisfying general types of recurrences.

Recently, we have extended algorithms for executing closure properties from D-finite functions (satisfying linear recurrence relations with polynomial coefficients) to DD-finite functions (satisfying linear differential equations with D-finite function coefficients). This is joint work with Antonio Jiménez-Pastor.

In this talk, we want to present different applications where some of these methods have been successfully applied without going into detail about the underlying algorithmic aspects.

What is . . . a Riemann-Hilbert problem?

Thomas Bothner
(*King's College London, UK*)
Time: Wednesday 24.07., 09:00, Room AM

Abstract: In its classical setting, the Riemann-Hilbert problem refers to Hilbert's 21st problem of constructing a Fuchsian ODE system with prescribed poles and a given monodromy group. Using singular integral equation techniques, Plemelj presented a solution to this problem in 1908 which became widely accepted. However, Kohn, Arnold and Il'yashenko noticed in the mid 1980s that Plemelj had actually worked on a problem similar to Hilbert's 21st for so-called regular ODE systems rather than Fuchsian ones. These new investigations resulted eventually in a negative answer to Hilbert's original problem given by Bolibruch in 1989 with further developments by Bolibruch and Kostov soon after.

Tangentially to the solution of Hilbert's classical problem, the singular integral equation techniques used therein, a.k.a. analytic factorizations of given functions defined on curves, gave rise to a class of modern Riemann-Hilbert factorization problems. In fact nowadays we view such problems as part of a broad analytical toolbox that is useful in the analysis of problems in mathematics and physics, for instance the Wiener-Hopf methods in hydrodynamics and diffraction. The goal of this talk is to first review some facts of the classical Riemann-Hilbert theory and then present a few recent developments of its modern counterpart. Special attention in the second part will be given to matrix- and operator-valued Riemann-Hilbert problems that arise in random matrix theory and integrable probability.

State revival in spin networks, graphs and orthogonal polynomials

Luc Vinet
(*Centre de Recherches Mathématiques (CRM), Université de Montréal, Canada*)
Time: Thursday 25.07., 09:00, Room AM

Abstract: This talk will offer a survey of old and new results on the revival of quantum states along spin networks. I shall concentrate on two protocols: 1. Perfect State Transfer (PST) that has a qbit transported from one site to another with probability one and 2. Fractional Revival (FR) at two sites which sees an initial state being reproduced periodically at two locations and is tantamount to entanglement generation. How to design networks that enact these processes is the basic question.

I shall focus first on chains or weighted paths and explain how this engineering issue amounts to inverse spectral problems where orthogonal polynomials play a central role. I shall stress that end-to-end PST requires that the associated Jacobi matrix be persymmetric or mirror-symmetric. Examples that involve the Krawtchouk, para-Krawtchouk, dual Hahn univariate polynomials will be discussed.

Association schemes will be brought to bear on the analysis. Here the connection with orthogonal polynomials will arise through the Bose-Mesner algebra. I shall explore in certain instances the possibility that the dynamics on paths admit lifts to quantum walks on graphs of certain schemes.

I shall further present a model identified recently and based on the $q = -1$ limit of the dual q-Hahn polynomials, that exhibits asymmetric PST over even sites and FR between the odd ones. We shall use the spectral theory of orthogonal polynomials to identify how mirror symmetry is replaced in this case.

With an eye to looking at PST and FR in two-dimensional spin networks, we shall discuss the unfamiliar ordered 2-Hamming scheme and uncover its connection to the bivariate Krawtchouk polynomials of Tratnik type. We shall finally indicate how a 2-dimensional spin network with interesting PST and FR properties can be identified from projecting a quantum walk on a weighted graph of this scheme onto the plane.

The talk will end with indications of additional mathematical and physical connections.

Based on work with: P.-A. Bernard (Montreal), M. Christandl (Copenhagen), G. Coutinho (Belo Horizonte), V. X. Genest (Boston), E. Loranger (Montreal), H. Miki (Tokyo), C. Tamon (Postdam, US), S. Tsujimoto (Kyoto), A. Zhedanov (Beijing), H. Zhan (Montreal)

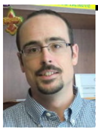
On orthogonal polynomials and the long-time behavior of completely integrable systems

Irina Nenciu
(*University of Illinois at Chicago, USA*)
Time: Thursday 25.07., 14:00, Room AM

Abstract: The study of any differential equation aims to reveal, in as much detail as possible, the behavior of solutions especially as time goes to infinity. While this is in most cases a lofty goal, it becomes much more approachable for completely integrable equations. These are, loosely speaking, nonlinear differential equations with a sufficiently large number of conserved quantities, and in their simplest, finite dimensional incarnation they are subject to the Liouville-Arnold-Jost (LAJ) theorem, which guarantees the existence of a global change of variables linearizing the flow. Thus completely integrable equations are much more approachable mathematically than non-integrable ones, while still exhibiting a much richer range of behaviors than linear differential equations do.

The catch in the program sketched above is that the LAJ Theorem, while absolutely remarkable, is not constructive. But in order to fully exploit the completely integrable structure of a differential equation, one needs to find the specific change of variables which realizes the linearization. The goal of my talk is three-fold. First, I will sketch the general theory of complete integrability. I will then focus on two concrete completely integrable partial difference equations, namely the Toda lattice and the (defocusing) Ablowitz-Ladik equations. In what is known as the finite case, finding the linearizing change of variables for both of these equations relies in a fundamental way on the theory of orthogonal polynomials on the real line and the unit circle, and I will spend most of the talk on clarifying this connection. Finally, I will consider these same equations in the periodic setting, where more subtle aspects of orthogonal polynomial theory are needed. Time permitting, I will conclude with some recent work and open questions related to the analysis of these cases.

Negative thinking and polynomial analogs



Armin Straub
(*University of South Alabama, USA*)
Time: Friday 26.07., 09:00, Room AM

Abstract: Gaussian binomial coefficients are at the heart of many other polynomial generalizations of interest in special functions, including the q-gamma function and basic hypergeometric series. We discuss the basic properties of Gaussian binomial coefficients and illustrate that many of the combinatorial, analytic and algebraic properties of the usual binomial coefficients permit natural polynomial extensions. On the arithmetic side, various classical congruences extend to the polynomial world. Examples include the famous Lucas congruences

$$\binom{n}{k} \equiv \binom{n_0}{k_0}\binom{n_1}{k_1}\cdots\binom{n_d}{k_d} \pmod{p},$$

where n_i and k_i are the p-adic digits of n and k, as well as Ljunggren's congruences, which state that $\binom{ap}{bp}$ is congruent to $\binom{a}{b}$ modulo p^3 for primes $p \geq 5$.

Through the gamma function, usual binomial coefficients conveniently extend to complex parameters. However, the gamma quotient definition and continuity do not immediately lead to well-defined values for negative integers. In the early 90s, however, Loeb showed that an appropriate natural extension of the binomial coefficients to negative (integer) entries continues to satisfy many of the fundamental properties. In particular, he gave a uniform binomial theorem as well as a combinatorial interpretation in terms of choosing subsets of sets with a negative number of elements. We tell this remarkable, yet little known, story and indicate that all of it can be extended to the case of Gaussian binomial coefficients.

We then discuss various open challenges in the context of supercongruences, series for $1/\pi$, and log-concavity, for which the q-point of view provides a valuable perspective. Time permitting, we will introduce a polynomial analog of the Apéry numbers, the famous sequence which underlies Apéry's proof of the irrationality of $\zeta(3)$. Together with their siblings, known as Apéry-like, they enjoy remarkable properties, including connections with modular forms, and have appeared in various contexts. One of their (still partially conjectural) properties is that these sequences satisfy supercongruences, a term coined by Beukers to indicate that the congruences are modulo exceptionally high powers of primes. We indicate how these congruences, which may be considered as generalizations of Ljunggren's congruences for the binomial coefficients, extend to the polynomial setting as well.

This talk is, in parts, based on joint work with Sam Formichella.

Rational solutions of Painlevé equations

Peter A. Clarkson
(*School of Mathematics, Statistics and Actuarial Science, University of Kent, Canterbury, UK*)
Time: Friday 26.07., 12:00, Room AM

Abstract: The six Painlevé equations, whose solutions are called the Painlevé transcendents, were derived by Painlevé and his colleagues in the late 19th and early 20th centuries in a classification of second order ordinary differential equations whose solutions have no movable critical points. In the 18th and 19th centuries, the classical special functions such as Bessel, Airy, Legendre and hypergeometric functions, were recognized and developed in response to the problems of the day in electromagnetism, acoustics, hydrodynamics, elasticity and many other areas. Around the middle of the 20th century, as science and engineering continued to expand in new directions, a new class of functions, the Painlevé functions, started to appear in applications. The list of problems now known to be described by the Painlevé equations is large, varied and expanding rapidly. The list includes, at one end, the scattering of neutrons off heavy nuclei, and at the other, the distribution of the zeros of the Riemann-zeta function on the critical line $\mathrm{Re}(z) = \frac{1}{2}$. Amongst many others, there is random matrix theory, the asymptotic theory of orthogonal polynomials, self-similar solutions of integrable equations, combinatorial problems such as the longest increasing subsequence problem, tiling problems, multivariate statistics in the important asymptotic regime where the number of variables and the number of samples are comparable and large, and also random growth problems.

The Painlevé equations possess a plethora of interesting properties including a Hamiltonian structure and associated isomonodromy problems, which express the Painlevé equations as the compatibility condition of two linear systems. Solutions of the Painlevé equations have some interesting asymptotics which are useful in applications. They possess hierarchies of rational solutions and one-parameter families of solutions expressible in terms of the classical special functions, for special values of the parameters. Further the Painlevé equations admit symmetries under affine Weyl groups which are related to the associated Bäcklund transformations.

In this talk I shall discuss rational solutions of Painlevé equations. Although the general solutions of the six Painlevé equations are transcendental, all except the first Painlevé equation possess rational solutions for certain values of the parameters. These solutions are usually expressed in terms of logarithmic derivatives of special polynomials that are Wronskians, often of classical orthogonal polynomials such as Hermite and Laguerre. It is also known that the roots of these special polynomials are highly symmetric in the complex plane. The polynomials arise in applications such as random matrix theory, vortex dynamics, in supersymmetric quantum mechanics, as coefficients of recurrence relations for semi-classical orthogonal polynomials and are examples of exceptional orthogonal polynomials.

In particular, I shall discuss rational solutions of the $A_2^{(1)}$-Painlevé system

$$\begin{aligned} f_0' + f_0(f_1 - f_2) &= \alpha_0, \\ f_1' + f_1(f_2 - f_0) &= \alpha_1, \\ f_2' + f_2(f_0 - f_1) &= \alpha_2, \end{aligned}$$

where $' \equiv \mathrm{d}/\mathrm{d}z$, with α_0, α_1 and α_2 constants, which is equivalent to the fourth Painlevé equation, and describe some new rational solutions of the $A_4^{(1)}$-Painlevé system

$$\begin{aligned} f_0' + f_0(f_1 - f_2 + f_3 - f_4) &= \alpha_0, \\ f_1' + f_1(f_2 - f_3 + f_4 - f_0) &= \alpha_1, \\ f_2' + f_2(f_3 - f_4 + f_0 - f_1) &= \alpha_2, \\ f_3' + f_3(f_4 - f_0 + f_1 - f_2) &= \alpha_3, \\ f_4' + f_4(f_0 - f_1 + f_2 - f_3) &= \alpha_4, \end{aligned}$$

with $\alpha_0, \alpha_1, \ldots, \alpha_4$ constants, which is the second member of the hierarchy.

MS01: Orthogonal polynomials, special functions, and functional equations

Organizers: Walter van Assche (*Katholieke Universiteit Leuven, Belgium*)
Galina Filipuk (*University of Warsaw, Poland*)
Yoshishige Haraoka (*Kumamoto University, Japan*)

In this mini-symposium we would like to gather together experts on linear special functions (e.g., hypergeometric) and nonlinear special functions (e.g., Painlevé equations) to discuss different aspects of these functions and recent advances in differential, difference and q-difference equations. One aspect is the relation between orthogonal polynomials and the Painlevé equations and the interest will be in discrete Painlevé equations and special solutions of the Painlevé differential equations that appear in the analysis of orthogonal polynomials.

Invariants of difference equations and transformation formulae for hypergeometric functions

01.01 **Akihito Ebisu**
(*Faculty of Information and Computer Science, Chiba Institute of Technology, Japan*)
Time: Tuesday 23.07., 15:30 - 16:00, Room AM

Abstract: We introduce invariants of linear difference equations. Moreover, using these invariants, we develop a systematic method for constructing transformation formulae for hypergeometric functions. By applying this method to hypergeometric functions, not only known formulae, such as algebraic transformation formulae and transformation formulae of special values, but also new-type transformation formulae are obtained.

Discrete Laplace method and hypergeometric continued fractions

01.02 **Katsunori Iwasaki**
(*Hokkaido University, Japan*)
Time: Tuesday 23.07., 16:00 - 16:30, Room AM

Abstract: A discrete analogue of Laplace's method for hypergeometric series containing a large parameter is developed. It aims to calculate large parameter asymptotics of the series by regarding the sum as a "discrete integral". It is particularly useful when Euler's integral representation of the series diverges so that the classical (continuous) Laplace method for integrals is not available. As an application leading asymptotics of the truncation error for Gauss's ${}_2F_1$ continued fraction is determined exactly, as well as for an infinite number of ${}_3F_2(1)$ continued fractions. Some discussions are also made about contiguous relations from which hypergeometric continued fractions are derived. This is in part a joint work with Akihito Ebisu. A more detailed account can be found in arXiv:1904.03350v1, Ramanujan J. 49 (2019), no.1, 159-213, and J. Math. Anal. Appl. 463 (2018), no.2, 593-610.

Some special functions of matrix integral type and quantum Painlevé equations

01.03 **Hironobu Kimura**
(*Graduate school of Science and Technology, Kumamoto University, Japan*)
Time: Tuesday 23.07., 16:30 - 17:00, Room AM

Abstract: We consider several special functions defined by matrix integrals on Hermitian matrix space, namely, analogues of Gauss hypergeomeric, Kummer's confluent hypergeometric, Bessel, Hermite-Weber and Airy function. We would like to discuss the relation of these functions of matrix integral type with some semi-classical orthogonal polynomials and with the polynomial solutions of quantum Painlevé equations.

Asymptotic analysis for a confluent KZ type equation

01.04 **Yoshishige Haraoka**
(*Department of Mathematics, Kumamoto University, Japan*)
Time: Tuesday 23.07., 17:00 - 17:30, Room AM

Abstract: We are interested in the asymptotic analysis of integrable connections of irregular singular type in several variables. Majima (LNM 1075, Springer, 1984) gave a fundamental idea of asymptotic expansion in several variables, and developed a general theory. However, there seems no essential example where Majima's asymptotic expansion is calculated. The asymptotic analysis in several variables seems to be difficult because there were few examples of integrable connections. In applying the Katz theory on rigid local systems, we get a way of constructing integrable connections in a recursive way, and can obtain infinitely many examples. For example, we obtain a confluent KZ type equation from an integrable connection satisfied by Appell's hypergeometric series F_4. We show how we can get Majima's asymptotic expansions for the connection.

More precise symmetric descriptions for properties of the Askey-Wilson Polynomials and their symmetric sub-families

01.05 **Howard Cohl**
(*National Institute of Standards and Technology, Gaithersburg, Maryland, USA*)
Time: Wednesday 24.07., 10:30 - 11:00, Room AM

Abstract: We give a more precise symmetric parametric description for various properties of the Askey-Wilson polynomials including hypergeometric and q-integral representations. This description is then applied to connection relations and to generating functions. We produce new q-integrals and basic hypergeometric transformations when applied to known generating functions for the symmetric families. This method is applied to produce generating functions and generalized generating functions in the two regimes of q and $1/q$ for $0 < |q| < 1$. This is joint work with Roberto S. Costas-Santos.

On algebraic independence of solutions for systems of algebraic Mahler functional equations

01.06 **Hiroshi Ogawara**
(*Kumamoto University, Japan*)
Time: Wednesday 24.07., 11:00 - 11:30, Room AM

Abstract: Many studies of functional equations of Mahler type have been conducted for algebraic independence of their solutions and algebraic independence of their special values. In this talk, we give criteria for algebraic independence of solutions for a new class of systems of algebraic Mahler functional equations. As an application, we show algebraic independence of the solutions of infinite product forms. We will speak of algebraic independence of special values of our solutions and asymptotic analysis of Mahler functions by using perturbation methods.

Connection problem of GKZ hypergeometric functions

01.07 **Saiei-Jaeyeong Matsubara-Heo**
(*Kobe University, Japan*)
Time: Wednesday 24.07., 11:30 - 12:00, Room AM

Abstract: GKZ (Gelfand, Kapranov, Zelevinsky) system is a holonomic system which describes classical hypergeometric systems in a unified manner. The study of GKZ system is, as a principle, controlled by the combinatorics of the Newton polytope. A typical manifestation of such a mechanism is the description of a connection problem of GKZ hypergeometric functions between "nearby toric infinities". It is expected

that the resulting connection formulae give a method of computing the monodromy group. In this talk, we formulate the connection problem for series solutions when GKZ system is regular holonomic. We give an explicit connection formula described by combinatorics of regular triangulations of the Newton polytope.

Discrete Painlevé Equations in Tiling Problems

01.08 **Anton Dzhamay**
(*School of Mathematical Sciences, University of Northern Colorado, Greeley, USA*)
Time: Wednesday 24.07., 12:00 - 12:30, Room AM

Abstract: The notion of a *gap probability* is one of the main characteristics of a probabilistic model. In [4] A. Borodin, extending to the discrete case a well-known relationswhip between gap probabilities and differential Painlevé equations, showed that for some discrete probabilistic models of Random Matrix Type discrete gap probabilities can be expressed through solutions of discrete Painlevé equations. Many examples of such correspondence for the gap probabilities of discrete polynomial ensembles are listed in [2] and are based on the formalism of discrete Riemann-Hilbert problem and its connection with the isomonodromic deformations of d-connections on vector bundles, [1].

In this work we generalize the results of A. Knizel [5] and consider a probabilistic model of random surfaces known as boxed plane partitions (or, equivalently, of lozenge tilings of a hexagon). When equipped with the uniform distribution, this model can be connected with the *Hahn* discrete orthogonal polynomial ensemble. In [3] A. Borodin, V. Gorin, and E. Rains proposed a very general multi-parameter probability weights for such partitions that, in certain limits, map to the q-Racah discrete polynomial ensemble, and from that to Racah, q-Hahn, and Hahn ensembles. This degeneration matches the degeneration in the classification scheme for discrete Painlevé equation that is due to H. Sakai, [7], as shown below. One of our goals is to understand this matching, including the degenerations.

We begin by describing the moduli space of q-connections that correspond to the q-Racah probability distribution and match it with the Space of Initial Conditions of discrete Painlevé equations of surface type $A_1^{(1)}$ and symmetry type $E_7^{(1)}$. Then, using the geometric techniques of Sakai's theory, such as the Period Map and the Birational Representation of Affine Weyl Groups, we show how to find a highly non-trivial change of variables from the original spectral (isomonodromic) coordinates to the Painlevé coordinates in which our dynamics matches the standard dynamics as written in [6]. This enables the effective computation of discrete gap probabilities for this model. Further, this change of variables is compatible with the parameter degeneration in both the weight degeneration cascade and the discrete Painlevé degeneration cascade. Finally, the isomonodromic problem gives is a new symmetric Lax pair for the q-P$(A_1^{(1)})$ equation.

[1] D. Arinkin and A. Borodin, *Moduli spaces of d-connections and difference Painlevé equations*, Duke Math. J. **134** (2006), no. 3, 515–556.

[2] Alexei Borodin and Dmitriy Boyarchenko, *Distribution of the first particle in discrete orthogonal polynomial ensembles*, Comm. Math. Phys. **234** (2003), no. 2, 287–338.

[3] Alexei Borodin, Vadim Gorin, and Eric M. Rains, *q-distributions on boxed plane partitions*, Selecta Math. (N.S.) **16** (2010), no. 4, 731–789.

[4] Alexei Borodin, *Discrete gap probabilities and discrete Painlevé equations*, Duke Math. J. **117** (2003), no. 3, 489–542.

[5] Alisa Knizel, *Moduli spaces of q-connections and gap probabilities*, International Mathematics Research Notices (2016), no. 22, 1073–7928.

[6] Kenji Kajiwara, Masatoshi Noumi, and Yasuhiko Yamada, *Geometric aspects of Painlevé equations*, J. Phys. A **50** (2017), no. 7, 073001, 164.

[7] Hidetaka Sakai, *Rational surfaces associated with affine root systems and geometry of the Painlevé equations*, Comm. Math. Phys. **220** (2001), no. 1, 165–229.

Orthogonal polynomials with ultra-exponential weight functions: an explicit solution to the Ditkin-Prudnikov problem

01.09 **Semyon Yakubovich**
(*Faculdade da Ciências, Universidade do Porto, Portugal*)
Time: Thursday 25.07., 10:30 - 11:00, Room AM

Abstract: In this talk we give an interpretation of new sequences of orthogonal polynomials with ultra-exponential weight functions in terms of the so-called composition orthogonality. The 3-term recurrence relations, explicit representations, generating functions and Rodrigues-type formulae are derived. The method is based on differential properties of the involved special functions and their representations in terms of the Mellin-Barnes and Laplace integrals. Certain advantages of the composition orthogonality are shown to find a relationship with the corresponding multiple orthogonal polynomial ensembles.

Asymptotics of solutions to the second Painlevé hierarchy

01.10 **Weiying Hu**
(*Department of Mathematics, City University of Hong Kong*)
Time: Thursday 25.07., 11:00 - 11:30, Room AM

Abstract: In this talk I will introduce the asymptotics of Hastings-McLeod solutions and Ablowitz-Segur solutions as $x \to \pm\infty$ for any α, the parameter appears in the equations. Moreover, the connection formulas for the Ablowitz-Segur solutions as $x \to \pm\infty$ are also derived. The method I use is the Deift and Zhou nonlinear steepest descent approach based on Riemann-Hilbert problem.

On some families of exactly solvable Schrödinger operators

01.11 **Jan Derezinski**
(*University of Warsaw, Faculty of Physics, Poland*)
Time: Thursday 25.07., 11:30 - 12:00, Room AM

Abstract: I will discuss various realizations of 1-dimensional Schrödinger operators with $1/x^2$ and $1/x$ potentials as closed operators on $L^2[0,\infty[$. Their resolvents can be expressed in terms of various kinds of Bessel and Whittaker functions. It is natural to organize them into holomorphic families, allowing for complex coupling constants. Their properties are sometimes quite surprising.

Exceptional extensions of some $q = -1$ classical orthogonal polynomials

01.12 **Yu Luo**
(*Graduate School of Informatics, Kyoto University, Japan*)
Time: Thursday 25.07., 12:00 - 12:30, Room AM

Abstract: In recent years, significant progress on exceptional orthogonal polynomial systems has been made by researchers from mathematical and physical aspects. It was proved that every system of exceptional orthogonal polynomials can be obtained from a classical orthogonal polynomial system through a sequence of Darboux transformations. Note that the classical orthogonal polynomials mentioned in this context are the Hermite, Laguerre and Jacobi polynomials, sometimes also be referred to as the "very" classical orthogonal polynomials. In general, polynomials in the Askey-Wilson scheme all can be called classical. The term classical means that apart from a three-term recurrence relation, these polynomials satisfy also an eigenvalue equation. Recently, several new families of polynomial systems which appear by taking a nontrivial limit $q = -1$ on orthogonal polynomials from the Askey-Wilson scheme have been identified classical. They satisfy eigenvalue problems with differential/difference operators of Dunkl type. Specifically, these polynomials are the Bannai-Ito polynomials, the big -1-Jacobi and the little -1-Jacobi polynomials. Unlike the previous cases, the associated Dunkl-type operators are of first-order which cannot

be factorized into two first-order as it was performed in an ordinary Darboux transformation. Therefore, we apply a generalized Darboux transformation by making use of a pair of intertwining relations satisfied by the Dunkl-type operators. In this way we derive the exceptional extensions of these $q = -1$ polynomials. An interesting fact of these exceptional orthogonal polynomial systems is that in several cases the corresponding degree sequences are consist of even numbers only, for example, $\{0, 2, 2, 4, 4, \dots\}$. We further study their ladder operators and the associated algebraic relations to address this fact. This is joint work with Satoshi Tsujimoto, Luc Vinet, and Alexei Zhedanov.

The Matrix Bochner Problem

01.13 **William Riley Casper**
(*Louisiana State University, Baton Rouge, Louisiana, USA*)
Time: Thursday 25.07., 15:30 - 16:00, Room AM

Abstract: We present a solution of the matrix Bochner problem, a long-standing open problem in the theory of orthogonal polynomials, with applications to diverse areas of research including representation theory, random matrices, spectral theory, and integrable systems. Our solution is based on ideas applied by Krichever, Mumford, Wilson and others, wherein the algebraic structure of an algebra of differential operators influences the values of the operators in the algebra. By using a similar idea, we convert the matrix Bochner problem to one about noncommutative algebras of GK dimension 1 which are module finite over their centers. Then the problem is resolved using the representation theory of these algebras.

Wronskian Appell polynomials and symmetric functions

01.14 **Niels Bonneux**
(*Katholieke Universitait Leuven, Belgium*)
Time: Thursday 25.07., 16:00 - 16:30, Room AM

Abstract: We study Wronskians of Appell polynomials indexed by integer partitions. These families of polynomials appear in rational solutions of certain Painlevé equations and in the study of exceptional orthogonal polynomials. We determine their derivatives, their average and variance with respect to Plancherel measure, and introduce several recurrence relations. Our proofs all exploit strong connections with the theory of symmetric functions.

Coefficients of Wronskian Hermite polynomials

01.15 **Marco Stevens**
(*Katholieke Universitait Leuven, Belgium*)
Time: Thursday 25.07., 16:30 - 17:00, Room AM

Abstract: Wronskians of Hermite polynomials appear in the rational solutions of Painlevé equations and in the field of exceptional orthogonal polynomials. In previous work, we exploited the combinatorial framework of these polynomials to derive recurrence relations in terms of integer partitions. In this talk, we use this framework to relate the coefficients of Wronskian Hermite polynomials to 2-cores and 2-quotients of integer partitions, as well as to characters of irreducible representations of the symmetric group. Joint work with Niels Bonneux (KU Leuven) and Clare Dunning (University of Kent).

Exceptional orthogonal polynomials from the semi-classical point of view

01.16 **Roberto S. Costas Santos**
(*Universidad de Alcalá, Madrid, Spain*)
Time: Thursday 25.07., 17:00 - 17:30, Room AM

Abstract: In this talk we briefly discuss these the different types of Laguerre and Jacobi exceptional polynomials and obtain some algebraic properties for these families by using certain first order differential

operators. We connect these families with certain semiclassical orthogonal polynomials. This is joint work with Jessica Stewart Kelly.

Nielsen's beta-function and some infinitely divisible distributions

01.17 **Henrik Laurberg Pedersen**
(*Department of Mathematical Sciences, University of Copenhagen, Denmark*)
Time: Friday 26.07., 10:30 - 11:00, Room AM

Abstract: Nielsen's beta-function is a classical special function related to Euler's gamma function. It is by definition a completely monotonic function. We obtain that it is a so-called logarithmically completely monotonic function, and determine the corresponding family of infinitely divisible distributions. This is based on the Steutel-Kristiansen theorem, relating generalized Stieltjes functions of positive order with logarithmically completely monotonic functions.

These results are related to Laplace transforms of positive, even and periodic functions. A method supplying us with a number of concrete examples of logarithmically completely monotonic functions is described.

The talk is based on joint work with Christian Berg (University of Copenhagen) and Stamatis Koumandos (University of Cyprus).

On the Heun functions

01.18 **Galina Filipuk**
(*University of Warsaw, Poland*)
Time: Friday 26.07., 10:30 - 11:00, Room HS 5

Abstract: In this talk I shall present some properties of the Heun functions.

Recurrence equations and their classical orthogonal polynomial solutions on a quadratic or a q-quadratic lattice

01.19 **Daniel Duviol Tcheutia**
(*Institute of Mathematics, University of Kassel, Germany*)
Time: Friday 26.07., 11:00 - 11:30, Room AM

Abstract: If $(p_n(x))_{n\geq 0}$ is an orthogonal polynomial system, then $p_n(x)$ satisfies a three-term recurrence relation of type

$$p_{n+1}(x) = (A_n x + B_n)p_n(x) - C_n p_{n-1}(x) \quad (n = 0, 1, 2, \ldots, p_{-1} \equiv 0),$$

with $C_n A_n A_{n-1} > 0$. On the other hand, Favard's theorem states that the converse is true. A general method to derive the coefficients A_n, B_n, C_n in terms of the polynomial coefficients of the divided-difference equations satisfied by orthogonal polynomials on a quadratic or q-quadratic lattice is recalled. If a three-term recurrence relation is given as input, the Maple implementations `rec2ortho` of Koorwinder and Swarttouw (1996) or `retode` of Koepf and Schmersau (2002) can identify its solution which is a (linear transformation of a) classical orthogonal polynomial system of a continuous, a discrete or a q-discrete variable, if applicable. The two implementations `rec2ortho` and `retode` do not handle classical orthogonal polynomials on a quadratic or q-quadratic lattice. Motivated by an open problem, submitted by Alhaidari during the 14th International Symposium on Orthogonal Polynomials, Special Functions and Applications, which will serve as application, the Maple implementation `retode` of Koepf and Schmersau is extended to cover classical orthogonal polynomial solutions on quadratic or q-quadratic lattices of three-term recurrence relations.

On connection problem of q-conformal blocks and its application

01.20 **Hajime Nagoya**
(*Kanazawa University, Ishikawa, Japan*)
Time: Friday 26.07., 11:30 - 12:00, Room AM

Abstract: Our q-conformal block function is derived from an expectation value of intertwiners between Fock modules of Ding-Iohara-Miki algebra. It is a series with explicit coefficients and can be identified with the q-Nekrasov partition function. I will explain how we solve the connection problem of the q-conformal block with the degenerate condition. Using the connection formula, I construct a fundamental solution of a q-linear difference system and obtain tau functions of a q-Painlevé system whose limit is the Fuji-Suzuki-Tsuda system.

MS02: Hypergeometric functions

Organizer: Diego Dominici (*Johannes Kepler University Linz, Austria*)

In this mini-symposium, we will consider recent developments in the field of hypergeometric functions, including generalized hypergeometric functions, basic hypergeometric functions and elliptic hypergeometric functions. Topics will cover summation formulas, asymptotic expansions, integrals and series, etc.

Large parameter asymptotics for hypergeometric and Legendre functions

02.01 **Adri B. Olde Daalhuis**
(*University of Edinburgh, UK*)
Time: Monday 22.07., 10:30 - 11:00, Room AM

Abstract: Surprisingly, apart from some special cases, simple asymptotic expansions for the associated Legendre functions $P_\nu^\mu(z)$ and $Q_\nu^\mu(z)$ for large degree ν or large order μ are not available in the literature. In this presentation we will fill this gap by deriving several simple (inverse) factorial expansions for these functions and provide sharp and realistic bounds on their error terms. In the cases that ν is an integer or 2μ is an odd integer, many of these new expansions terminate and provide finite representations in terms of simple functions. Most of these representations appear to be new.

It is well known that the hypergeometric series can be regarded as a large-c asymptotic expansion for the hypergeometric function $F(a, b; c; z)$. We will also present computable bounds for the remainder term of this expansion. To our best knowledge, no such estimates have been given in the literature.

Monodromy of the generalized hypergeometric equation in the Frobenius basis

02.02 **Leslie Molag**
(*Katholieke Universitait Leuven, Belgium*)
Time: Monday 22.07., 11:00 - 11:30, Room AM

Abstract: In his 1961 Ph.D. thesis Levelt gave an explicit construction for determining the monodromy group of the generalized hypergeometric equation. His result yields the explicit form of the monodromy matrices in a specific basis, which turns out to be connected to Mellin-Barnes integrals. In some situations we would like to know the form of these matrices in a different basis though, especially in the arguably most standard basis: the Frobenius basis. To obtain the form of the monodromy matrices in this basis I have followed an approach set out by Beukers. A particular challenge is the maximally unipotent case, where logarithmic terms turn up in the Frobenius basis. The emphasis of this talk will be on elucidating the general explicit form of the monodromy matrices for this case.

New series representation formulas for modified Bessel function of second kind of integer order

02.03 **Dragana Jankov Maširević**
(*Josip Juraj Strossmayer University of Osijek, Croatia*)
Time: Monday 22.07., 11:30 - 12:00, Room AM

Abstract: The main aim of this talk is to present two finite sum representation formulas for the modified Bessel function of the second kind K_n of positive integer order $n \in \mathbb{N}$: one of them including, among K_0, K_1, the generalized hypergeometric function ${}_1F_2$ and another which includes only K_0 and the modified Bessel functions of the first kind I_0 and I_1. Also, the obtained finite sum representations are superior with respect to the known expression in a computational efficiency examination, which will be illustrated on several examples.

Newton diagram for the positivity of ${}_1F_2$ hypergeometric functions and Askey-Szegő problem

02.04 **Yong-Kum Cho**
(*Chung-Ang University, Seoul, South Korea*)
Time: Monday 22.07., 12:00 - 12:30, Room AM

Abstract: Concerning the positivity inequality

$$\text{(P)} \qquad {}_1F_2\left[\begin{matrix} a \\ b, c \end{matrix} \,\middle|\, -\frac{x^2}{4}\right] \geq 0 \qquad (x > 0),$$

with $a > 0$ fixed, we prove that if the parameter pair (b, c) belongs to certain hyperbolic region in $\mathbb{R}_+^2$ containing the Newton diagram associated to $\{(a + 1/2, 2a), (2a, a + 1/2)\}$, then (P) holds true. As an application, we consider the Askey-Szegő problem, related with

$$\int_0^x t^{-\beta} J_\alpha(t)dt \geq 0 \qquad (x > 0),$$

for which the best possible range of parameters is known in an implicit formulation involving transcendental equations, and obtain the lower and upper bounds for this range of parameters. In addition, we apply our criteria to improve the positivity region for the Lommel functions established by J. Steinig in 1972.

[1] Y.-K. Cho and S.-Y. Chung, *On the positivity and zeros of Lommel functions: Hyperbolic extension and interlacing*, J. Math. Anal. Appl. 470, pp. 898–910 (2019)
[2] Y.-K. Cho, S.-Y. Chung and H. Yun, *An extension of positivity for integrals of Bessel functions and Buhmann's radial basis functions*, Proc. Amer. Math. Soc., Series B, Vol. 5, pp. 25–39 (2018)
[3] Y.-K. Cho, S.-Y. Chung and H. Yun, *Rational extension of the Newton diagram for the positivity of ${}_1F_2$ hypergeometric functions and Askey-Szegő problem*, arXiv:1805.11855, Constr. Approx. (to appear) (2019)
[4] Y.-K. Cho and H. Yun, *Newton diagram of positivity for ${}_1F_2$ generalized hypergeometric functions*, Integral Transforms Spec. Funct. 29, pp. 527–542 (2018)

Symbolic computation for D^n-finite functions

02.05 **Antonio Jiménez Pastor**
(*Research Institute for Symbolic Computation, Johannes Kepler University, Linz, Austria*)
Time: Monday 22.07., 15:30 - 16:00, Room AM

Abstract: Hypergeometric functions fall naturally into the category of D-finite (or holonomic) functions, being able to obtain a linear differential equation for each ${}_pF_q$ hypergeometric function. We recently extended the concept of D-finite functions (power series satisfying linear differential equations with polynomial coefficients) to the recursive idea of D^n-finite functions (power series satisfying linear differential equation with D^{n-1}-finite coefficients. We will show in this talk the definition of these D^n-finite power series and the main properties they satisfy, providing combinatorial examples and open questions related with hypergeometric functions.

Hypergeometric form of fundamental theorem of calculus

02.06 **Petr Blaschke**
(*Silesian University in Opava, Czech Republic*)
Time: Monday 22.07., 16:00 - 16:30, Room AM

Abstract: We introduce a natural method of computing antiderivatives of a large class of functions which stems from the observation that the series expansion of an antiderivative differs from the series expansion of the corresponding integrand by just two Pochhammer symbols. All antiderivatives are thus, in a sense, "hypergeometric". And hypergeometric functions are therefore the most natural functions to integrate.

In this talk we would like to make two points: First, the method presented is *easy*. So much so that it can be taught in undergraduate university level. And second: It may be used to prove some of the more challenging examples.

In particular, we show that

$$\int_0^\infty \sqrt[8]{\frac{x^2+8x+8-4(2+x)\sqrt{1+x}}{x^{11}}}\mathrm{d}x = \frac{4\Gamma^2\left(\frac{1}{4}\right)}{3\sqrt{2-\sqrt{2}}\sqrt{\pi}},$$

$$G = \Re\left({}_3F_2\left(\begin{matrix}1 & 1 & 1\\ & 2 & 2\end{matrix};\mathrm{i}\right)\right) = \Im\left(\left[\epsilon^2\right]{}_2F_1\left(\begin{matrix}\epsilon & \epsilon\\ & 1\end{matrix};\mathrm{i}\right)\right) = \frac{1}{8}\left(\psi'\left(\frac{1}{4}\right)-\pi^2\right),$$

where G is the Catalan's constant and

$$\int_0^1 x\ln\frac{1}{1+x^2}K(\mathrm{i}x)\mathrm{d}x = \frac{1}{4\sqrt{2\pi}}\left((2-\ln 2)\Gamma^2\left(\frac{1}{4}\right)+4(\ln 2-4)\Gamma^2\left(\frac{3}{4}\right)\right),$$

where K is the complete elliptic integral of the first kind. All of this using a single technique.

A q-Hurwitz zeta function associated with a q-analogue of Bernoulli polynomials and numbers

02.07 **Zeinab Mansour**
(*Cairo University, Giza, Egypt*)
Time: Monday 22.07., 16:30 - 17:00, Room AM

Abstract: Ismail and Mansour in 2018 introduced a pair of q-analogue of the Bernoulli polynomials through the generating function

$$\frac{e_q(xt)}{e_q(t/2)E_q(t/2)-1} = \sum_{k=0}^\infty b_k(x;q)\frac{t^k}{[k]!}$$

$$\frac{E_q(xt)}{e_q(t/2)E_q(t/2)-1} = \sum_{k=0}^\infty B_k(x;q)\frac{t^k}{[k]!}$$

In this talk we introduce a q-analogue of the Hurwitz-Zeta function and Zeta function and prove that the q-zeta function satisfy the identities

$$\zeta_q(s) = \sum_{k=1}^\infty \xi_k^{s-1}\frac{\operatorname{Cos}\xi_k}{(\operatorname{Sin}_q\xi_k)'}, \qquad \zeta_q(-n) = \frac{B_n(q)}{[n+1]}, \quad n\in\mathbb{N}_0$$

where ξ_k are the positive zeros of the $\operatorname{Sin}_q z = \dfrac{(-iz(1-q);q)_\infty-(iz(1-q);q)_\infty}{2i}$, $B_n(q) = B_n(0;q) = b_n(0;q)$, and $[k] := \frac{1-q^k}{1-q}$. We also extend the results of Lidstone expansions introduced introduced by Ismail and Mansour in [Analysis and Applications, https://doi.org/10.1142/S0219530518500264] for Lidstone expansions.

Functional inequalities for the generalized Wright functions

02.08 **Sourav Das**
(*National Institute of Technology, Jamshedpur, Jharkhand, India*)
Time: Monday 22.07., 17:00 - 17:30, Room AM

Abstract: In this work, a generalization of Wright functions is considered. We derive some man value inequalities for this generalized function, such as Turán-type inequalities, Lazarević-type inequalities, Wilker-type inequalities and Redheffer-type inequalities. Furthermore, we establish monotonicity of ratios for sections of series of these generalized Wright functions, the obtained result is also closely related to Turán-type inequalities. Finally, some other related inequalities are also discussed as a consequence.

Sharp parameter range for interlacing of zeros of same degree Laguerre polynomials

02.09 **Kathy Driver**
(*University of Cape Town, South Africa*)
Time: Tuesday 23.07., 10:30 - 11:00, Room AM

Abstract: The sequence of Laguerre polynomials $\{L_n^{(\alpha)}(x)\}_{n=0}^{\infty}$ is orthogonal on $(0, \infty)$ with respect to the weight function $e^{-x}x^{\alpha}$ provided $\alpha > -1$. It is known that for each $n \in \mathbb{N}$, the zeros of $L_n^{(\alpha)}(x)$ and $L_n^{(\alpha+t)}(x)$ are interlacing for each t with $0 < t \leq 2$. We show that the t-interval $0 < t \leq 2$ is sharp in order for interlacing to hold for every $n \in \mathbb{N}$.

Hypergeometric transformations based on Hahn and Racah polynomials

02.10 **Robert S. Maier**
(*University of Colorado, Boulder, USA*)
Time: Tuesday 23.07., 11:00 - 11:30, Room AM

Abstract: Much as the Gauss hypergeometric function ${}_2F_1$ satisfies many transformation identities, the function ${}_3F_2$ can be quadratically and cubically transformed. For example, a ${}_3F_2$ with a parametric excess equal to $\frac{1}{2}$ or $-\frac{1}{2}$ may be quadratically transformed to a well-poised ${}_3F_2$ or a very well-poised ${}_4F_3$. Summation identities can be derived from such transformations by the classical technique of equating coefficients, or by Gessel–Stanton pairing. We show that the classical quadratic and cubic transformations of ${}_3F_2$ can extended: in the quadratic case, the parametric excess may be greater than $\frac{1}{2}$ or less than $-\frac{1}{2}$ by any natural number. The transformed functions now become hypergeometric functions of higher order, the added parameters of which make contact with the theory of orthogonal polynomials of a discrete argument. For instance, the added parameters can be the (negated) roots of certain dual Hahn or Racah polynomials, which are defined on a quadratic lattice; or in the cubic case, new polynomials with no evident orthogonal interpretation. Extended versions of summation identities of Whipple and Bailey can be derived from the extended transformations of ${}_3F_2$: for instance, extensions of Dougall's theorem on the sum of a 2-balanced, very well-poised ${}_7F_6(1)$.

A family of hypergeometric orthogonal polynomial sequences that contains all the families in the Askey scheme

02.11 **Luis Verde-Star**
(*Universidad Autónoma Metropolitana, Mexico City, Mexico*)
Time: Tuesday 23.07., 11:30 - 12:00, Room AM

Abstract: We introduce a family $\mathcal{H}$ of hypergeometric orthogonal polynomial sequences determined by three polynomials h, f and g with degrees at most 1, 2, and 3, respectively, where some coefficients of g depend on the coefficients of h and f. The sequences in $\mathcal{H}$ satisfy a generalized difference equation of order one.

We express the orthogonal polynomials using the Newton basis associated with the sequence $f(n)$, for $n \geq 0$, and for every sequence in $\mathcal{H}$ we find an explicit hypergeometric representation, the three-term recurrence relation, and the generalized moments with respect to the Newton basis. When f is constant we obtain the classical orthogonal sequences. When f is not constant the sequences satisfy a discrete orthogonality relation and we find the discrete weight function.

The recurrence coefficients for each one of the 15 families of polynomial sequences in the Askey scheme of hypergeometric orthogonal polynomials are obtained by direct substitution of particular values for the parameters in our general formulas.

A Bailey type factorization of Horn's H4 hypergeometric function

02.12 **Carlo Verschoor**
(*University of Utrecht, Netherlands*)
Time: Tuesday 23.07., 12:00 - 12:30, Room AM

Abstract: A well known identity by Bailey states that Appell's F4 function can be written as the product of two Gauss hypergeometric functions under a suitable specialization of the parameters. Other identities of this type are known for Appell's F2 and F3, which are closely related to Bailey's identity. The aim of this talk is to show that the same can be done for Horn's H4 function.

MS03: Trends on orthogonal polynomials in weighted Sobolev spaces

Organizers: Francisco Marcellán (*Universidad Carlos III de Madrid and ICMAT, Spain*)
Juan José Moreno-Balcázar (*Universidad de Almería, Spain*)

In this MS we will consider different approaches and applications of polynomials orthogonal with respect to inner products in weighted Sobolev spaces. The topics to be covered are interpolation and Fourier projectors and their applications to boundary value problems in one and several variables, asymptotic properties of such polynomials, distribution of zeros, convergence of Sobolev-Fourier expansions, Christoffel functions, moment problems, differential operators with Sobolev orthogonal polynomials as eigenfunctions, among others.

New trends on orthogonal polynomials in Sobolev spaces

03.01 **Francisco Marcellán and Juan J. Moreno–Balcázar**
(*Universidad Carlos III de Madrid and Universidad de Almería, Spain*)
Time: Monday 22.07., 15:30 - 16:00, Room HS 6

Abstract: In this presentation we will analyze some recent trends in the theory of orthogonal polynomials with respect to Sobolev inner products defined for vector of measures supported either on the real line or the unit circle respectively. Analytical and computational approaches will be discussed. In the first case we will focus the attention on asymptotic properties of such polynomials as well as the distribution of their zeros. In the second one, we will study questions related to boundary value problems for differential equations and Sturm-Liouville theory.

Operational methods in the study of Sobolev-Jacobi polynomials

03.02 **Nicolas Behr**
(*IRIF, Université Paris Diderot, France*)
Time: Monday 22.07., 16:00 - 16:30, Room HS 6

Abstract: Inspired by ideas from umbral calculus and based on the two types of integrals occurring in the defining equations for the gamma and the reciprocal gamma functions, respectively, we develop a multi-variate version of the so-called umbral image technique. Besides providing a class of new formulae for generalized hypergeometric functions and an implementation of series manipulations for computing lacunary generating functions, our main application of these Â techniques is the study of Sobolev-Jacobi polynomials. Motivated by applications to theoretical chemistry, we moreover present a deep link between generalized normal-ordering techniques introduced by Gurappa and Panigrahi, two-variable Hermite polynomials and our integral-based series transforms. This is joint work with G. Dattoli (ENEA Frascati), G.H.E. Duchamp (Paris 13), Silvia Licciardi (ENEA Frascati) and K.A. Penson (Paris 6).

Bispectral Laguerre and Jacobi type polynomials

03.03 **Manuel Domínguez de la Iglesia**
(*Universidad Nacional Autónoma de México, Mexico*)
Time: Monday 22.07., 16:30 - 17:00, Room HS 6

Abstract: We study the bispectrality of Laguerre and Jacobi type polynomials, which we define by taking linear combinations of a fixed number of consecutive Laguerre or Jacobi polynomials, respectively. These polynomials are eigenfunctions of higher-order differential operators and include, as particular cases, the Krall-Laguerre and the Krall-Jacobi polynomials. We show that these polynomials always satisfy higher-order recurrence relations (i.e., they are bispectral). We also prove that the Krall-Laguerre and the Krall-Jacobi families are the only Laguerre and Jacobi type polynomials which are orthogonal with respect to a measure in the real line. This is a joint work with Antonio J. Durán.

A new symmetric representation of the differential equation for the Laguerre-Sobolev polynomials

03.04 **Clemens Markett**
(*Aachen University of Technology, Germany*)
Time: Monday 22.07., 17:00 - 17:30, Room HS 6

Abstract: In the enduring fruitful research on orthogonal polynomials in weighted Sobolev spaces, the Laguerre-Sobolev polynomials are playing a prominent role. They are orthogonal with respect to a Sobolev-type inner product associated with the classical Laguerre measure on the positive half-line and two point masses $M, N > 0$ at the origin involving functions and their first derivatives. A particularly useful feature of the Laguerre-Sobolev polynomials is their property to arise as the eigenfunctions of a spectral differential operator which, for any Laguerre parameter $\alpha \in \mathbb{N}_0$, is of finite order $4\alpha + 10$. The main purpose of this talk is to establish a new symmetric form of this differential operator which consists of a number of elementary components depending on α, M, N. In particular, the new representation enables us to deduce the symmetry of the operator in the Sobolev space. This readily recovers the orthogonality of the polynomial eigenfunctions. The present results have strongly been motivated and guided by our recent observations that the higher-order differential equations for the so-called Bochner-Krall orthogonal polynomials possess an elementary symmetric form which differs considerably from the classical Lagrange symmetric form. Among them is the differential equation of order $2\alpha + 2\beta + 6$ for the generalized Jacobi polynomials which will serve us as a pattern to illustrate how the main features of the equation carry over to the Laguerre-Sobolev case. We close with some new promising results on the Jacobi-Sobolev equations.

Sobolev biorthogonal polynomials and the Gauss-Borel factorization

03.05 **Manuel Mañas**
(*Universidad Complutense de Madrid, Spain*)
Time: Tuesday 23.07., 10:30 - 11:00, Room HS 6

Abstract: We explore the Gauss-Borel description of the Sobolev birothogonality. A theory of deformations of Sobolev bilinear forms is proposed. We consider both polynomial deformations and a class of transformations related to the action of linear operators on the entries of a given bilinear form. Christoffel type formulae among new and old polynomial sequences are determined. We also discuss generalized Hankel symmetries.

"Hermitian + nilpotent" = Sobolev moment problem

03.06 **Franciszek Hugon Szafraniec**
(*Jagiellonian University, Kraków, Poland*)
Time: Tuesday 23.07., 11:00 - 11:30, Room HS 6

Abstract: I am going to raise the question assisted by the paper The Sobolev moment problem and Jordan dilations, *J. Math. Anal. Appl.* **444** (2016) 1675-1689 (with Michał Wojtylak). Some discussion toward the Dirichlet moment problem will be provided as well.

Fourier series of Sobolev polynomials for coherent pairs of Jacobi type

03.07 **Judit Mínguez**
(*Universidad de La Rioja, Spain*)
Time: Tuesday 23.07., 11:30 - 12:00, Room HS 6

Abstract: The study of orthogonal polynomials with respect to a Sobolev-type inner product has attracted the interest of many researchers in the last years as we can see in [3]. In [1] and [2], the authors proved convergence and uniform boundedness of the partial sums in some cases for Gegenbauer-Sobolev and Jacobi-Sobolev polynomials. In this work, we are going to study Sobolev orthonormal polynomals for

coherent pairs of measures of Jacobi type, in order to prove uniform boundedness and convergence for the partial sums. This is a joint work with O. Ciaurri.

[1] O. Ciaurri, J. Mínguez Ceniceros, Fourier series of Gegenbauer Sobolev Polynomials *SIGMA* **14** (2018), 024, 11 pages.
[2] O. Ciaurri, J. Mínguez Ceniceros, Fourier series of Jacobi-Sobolev polynomials, *Integral Transforms and Special Functions.* https://doi.org/10.1080/10652469.2018.1560279.
[3] F. Marcellán and Y. Xu, On Sobolev orthogonal polynomials, *Expo. Math.* **33** (2015), 308–352.

Rational approximation and Sobolev orthogonal polynomials

03.08 **Hector Pijeira Cabrera**
(*Universidad Carlos III de Madrid, Spain*)

Time: Tuesday 23.07., 12:00 - 12:30, Room HS 6

Abstract: Let $\{S_n\}_{n=0}^{\infty}$ be the sequence of orthogonal polynomialswith respect to the Sobolev-typeinner product

$$\langle f, g\rangle = \int_{-1}^{1} f(x)g(x)\, d\mu(x) + \sum_{j=1}^{N} \eta_j\, f^{(d_j)}(c_j)g^{(d_j)}(c_j),$$

where μ is in the Nevai class $\mathbf{M}(0,1)$, $\eta_j > 0$, $N, d_j \in \mathbb{Z}_+$ and $\{c_1, \dots, c_N\} \subset \mathbb{R} \setminus [-1,1]$. Under some restriction of order in the discrete part of $\langle\cdot,\cdot\rangle$, we proof that for n sufficiently large the zeros of S_n are real, simple, $n-N$ of them lie on $(-1,1)$ and each of the mass points c_j "attracts" one of the remaining N zeros. The sequences of associated polynomials $\{S_n^{[k]}\}_{n=0}^{\infty}$ are defined for each $k \in \mathbb{Z}_+$. We prove an analog of the Markov's theorem on rational approximation of some class holomorphic functions and we give an estimate of the "speed" of convergence. This is a joint work with Abel Díaz-González.

Eigenvalues of a differential operator related to classical discrete Sobolev orthonormal polynomials

03.09 **Juan F. Mañas-Mañas**
(*Universidad de Almería, Spain*)

Time: Tuesday 23.07., 15:30 - 16:00, Room HS 6

Abstract: We consider the discrete Sobolev inner product

$$(f,g)_S = \int f(x)g(x)d\mu + Mf^{(j)}(c)g^{(j)}(c), \quad j \in \mathbb{N}\cup\{0\}, \quad c \in \mathbb{R}, \quad M > 0,$$

where μ is a classical continuous measure with support on the real line (Jacobi, Laguerre or Hermite). The orthonormal polynomials with respect to this Sobolev inner product are eigenfunctions of a differential operator and obtaining the asymptotic behavior of the corresponding eigenvalues is the principal goal of this talk. This is a joint work with Juan J. Moreno-Balcázar.

On Freud-Sobolev type orthogonal polynomials: asymptotics and zeros

03.10 **Lino Gustavo Garza Gaona**
(*Physics and Mathematics Department, Universidad de Monterrey, Mexico*)

Time: Tuesday 23.07., 16:00 - 16:30, Room HS 6

Abstract: In this contribution we consider sequences of monic polynomials orthogonal with respect to the discrete Sobolev type inner product involving a quartic potential

$$\langle f,g\rangle_1 = \int_{\mathbb{R}} f(x)g(x)|x|^{2\lambda+1}e^{-x^4+tx^2}dx + M_0 f(0)g(0) + M_1 f'(0)g'(0).$$

In particular, we obtain algebraic properties related to their zeros, such as equations of motion with respect to the parameter t, and monotonicity results when M_0, M_1 tend to infinity. We also obtain some asymptotic properties for the coefficients on the recurrence relation that the Sobolev-type orthogonal polynomials satisfy.

On coherence relations between quasi-definite linear functionals and Sobolev orthogonal polynomials

03.11 **Luis Alejandro Molano Molano**
(*Universidad Pedagógica y Tecnológica de Colombia, Duitama, Colombia*)
Time: Tuesday 23.07., 16:30 - 17:00, Room HS 6

Abstract: In this talk we consider the non-coherence relation

$$\begin{aligned} &P_{n+1}^{[i]}(x) + a_n^{[1]} P_n^{[i]}(x) + a_n^{[2]} P_{n-1}^{[i]}(x) + b_n \left(Q_{n+1}(x) + c_n Q_n(x)\right) \\ &= \left(1 + b_n\right) R_{n+1}(x) + d_n R_n(x), \end{aligned} \tag{1}$$

where the sequences $\{P_n(x)\}_{n\geq 0}$, $\{Q_n(x)\}_{n\geq 0}$ and $\{R_n(x)\}_{n\geq 0}$ are orthogonal with respect to quasi-definite linear functionals u, v and w, respectively, with $P_k^{[i]}(x) := \frac{P_{k+i}^{(i)}(x)}{(k+1)_i}$, $i = 0, 1$, and $a_n^{[i]} b_n c_n d_n (1 + b_n) \neq 0$, $n \geq 0$. Furthermore the linear functionals u and v are related through the rational relation $\rho u = v$ where $\deg \rho > 1$. We pointed out that (1) is linked to the concept of *symmetric* $(1,1)-$*coherent pairs* and, under certain conditions on the linear functionals, it can become a coherence relation. Under such conditions, we analyze an inverse problem associated with (1) as well as a direct problem where $i = 1$, $a_n^{[2]}, c_n, d_n = 0$ and $b_n = 1$, for $n \geq 0$. Thereby we exhibit conditions under which the sequence $\{R_n(x)\}_{n\geq 0}$ is orthogonal with respect to a Borel positive measure μ supported on an infinite subset on the real line. The case when $\{Q_n(x)\}_{n\geq 0}$ and $\{P_n(x)\}_{n\geq 0}$ are the classical Chebyshev polynomials of the first and second kinds, respectively, is studied, as well as algebraic properties of the monic *Sobolev* polynomials, orthogonal with respect to the *Sobolev inner product*

$$\begin{aligned} \langle p, q \rangle_S \quad = \quad & \int_{-1}^{1} p(x)q(x)(1-x^2)^{-1/2} dx + \lambda_1 \int_{-1}^{1} p'(x)q'(x)(1-x^2)^{1/2} dx \\ & + \lambda_2 \int_{-1}^{1} p''(x)q''(x) d\mu(x), \end{aligned}$$

where $\lambda_1, \lambda_2 > 0$.

Sobolev orthogonal polynomials on the triangle

03.12 **Lidia Fernández**
(*Universidad de Granada, Spain*)
Time: Wednesday 24.07., 10:30 - 11:00, Room HS 6

Abstract: In [1], Yuan Xu generalized the standard bases of orthogonal polynomials on the triangle for negative values of the parameters and he showed Sobolev orthogonality for these polynomials. The purpose of this work is to analyze another family of mutually orthogonal polynomials on the triangle with respect to an inner product which involves some derivatives on one side of it. These polynomials are related with some Christoffel–Darboux transformation of one of the three term recurrence relations that polynomials on the triangle satisfy. Some algebraic and analytic properties will be deduced.

[1] Y. Xu, Approximation and orthogonality in Sobolev spaces on a triangle. *Constr. Approx.* **46** (2017) 349–434.

On Dunkl–Sobolev orthogonal polynomials in the ball involving reflection-invariant weights

03.13 **Leonardo Figueroa**
(*Universidad de Concepción, Chile*)
Time: Wednesday 24.07., 11:00 - 11:30, Room HS 6

Abstract: We investigate properties of spaces of polynomials orthogonal with respect to inner products in the ball involving $\mathbb{Z}_2^d$-invariant weights of the form

$$(1 - \|x\|^2)^{\kappa_{d+1}} \prod_{i=1}^{d} |x_i|^{\kappa_i}$$

and their associated Dunkl differential-difference operators. We deduce orthogonal decompositions of these spaces which then allow for characterizing them as eigenspaces of (weak) Sturm–Liouville-type operators, with approximation-theoretical consequences. This talk is based on joint work with Gonzalo A. Benavides.

Coherent pairs of bivariate orthogonal polynomials

03.14 **Misael Marriaga**
(*Universidad Rey Juan Carlos, Madrid, Spain*)
Time: Wednesday 24.07., 11:30 - 12:00, Room HS 6

Abstract: Coherent pairs of measures were introduced in 1991 and constitute a very useful tool in the study of Sobolev orthogonal polynomials on the real line. In this work, coherence and partial coherence in two variables appear as the natural extension of the univariate case. Given two families of bivariate orthogonal polynomials expressed as polynomial systems, they are a partial coherent pair if there exists a polynomials of the second family can be given as a linear combination of the first partial derivatives of (at most) three consecutive polynomials of the first family. A full coherent pair is a pair of families of bivariate orthogonal polynomials related by means of partial coherent relations in each variable. Consequences of this kind of relations concerning both families of bivariate orthogonal polynomials are studied.

MS04: Multivariate special functions related to Lie algebras

Organizer: Michael Schlosser (*Universität Wien, Vienna, Austria*)

In this mini-symposium recent developments on multivariate special functions related to Lie algebras, or root systems, will be considered. The topics include but are not restricted to symmetric functions (such as Macdonald polynomials, Macdonald-Koornwinder polynomials, etc.), integrable systems, related physical and combinatorial models, connections to representation theory, conformal field theory and character identities.

Intertwining operator for the dihedral group

04.01 **Yuan Xu**
(*University of Oregon, USA*)
Time: Thursday 25.07., 10:30 - 11:00, Room HS 6

Abstract: Dunkl operators associated to a dihedral group are a pair of differential-difference operators that generate a commutative algebra. The intertwining operator intertwines between this algebra and the algebra of ordinary differential operators. We will discuss an integral representation of the intertwining operator on a class of functions. As an application, closed formulas of the Poisson kernels are derived for sieved Gegenbauer polynomials and several related families of orthogonal polynomials.

The higher rank q-Bannai–Ito algebra and multivariate $(-q)$-Racah polynomials

04.02 **Hadewijch De Clercq**
(*Ghent University, Belgium*)
Time: Thursday 25.07., 11:00 - 11:30, Room HS 6

Abstract: The q-Racah polynomials are well-known to be bispectral, i.e. they can be defined through both a second-order q-difference equation and a three-term recursion relation. This bispectrality is described algebraically by the Askey–Wilson or Zhedanov algebra, and its counterpart under a transformation $q \to -q$, the so-called q-Bannai–Ito algebra. In this talk, I will explain how these connections can be generalized to multiple variables. We will construct a higher rank extension for the q-Bannai–Ito algebra by exploiting the Hopf algebraic structure of quantum groups. Then we will show how this novel algebra encodes the bispectrality of Gasper & Rahman's multivariate $(-q)$-Racah polynomials. More precisely, we will study how this algebra acts on the discrete series representation of the corresponding quantum group, and identify a class of canonical bases. Several such bases are in duality, in the sense that their overlap coefficients can be expressed as multivariate $(-q)$-Racah polynomials. Iliev's bispectral q-shift operators give rise to a discrete realization of the higher rank q-Bannai-Ito algebra. Finally, I will discuss the limit $q \to 1$, which suggests a construction for multivariate Bannai–Ito polynomials.

This is joint work with Hendrik De Bie.

A quantum algebra approach to multivariate Askey–Wilson polynomials

04.03 **Wolter Groenevelt**
(*Delft University of Technology, Netherlands*)
Time: Thursday 25.07., 11:30 - 12:00, Room HS 6

Abstract: In this talk we show that the multivariate Askey–Wilson polynomials introduced by Gasper and Rahman occur as matrix elements of representations of the quantum algebra $\mathcal{U}_q(su(1,1))$. From this interpretation several properties of the polynomials, e.g. orthogonality, can be obtained.

Elliptic extension of Gustafson's q-integral of type G_2

04.04 **Masahiko Ito**
(*University of the Ryukyus, Nishihara, Okinawa Prefecture, Japan*)
Time: Thursday 25.07., 12:00 - 12:30, Room HS 6

Abstract: I will present an evaluation formula for an elliptic beta integral of type G_2. The integral is expressed by a product of Ruijsenaars' elliptic gamma functions, and the formula includes that of Gustafson's q-beta integral of type G_2 as a special limiting case as $p \to 0$. The elliptic beta integral of type BC_1 by van Diejen and Spiridonov is effectively used in the proof of the evaluation formula. This is a joint work with M. Noumi.

Convolution identities arising from the Lie superalgebra $\mathfrak{osp}(1|2)$

04.05 **Erik Koelink**
(*Radboud University, Nijmegen, Netherlands*)
Time: Thursday 25.07., 15:30 - 16:00, Room HS 6

Abstract: There is a link between various sets of orthogonal polynomials and the representation theory of the Lie superalgebra $\mathfrak{osp}(1|2)$. We consider the irreducible unitary representations of $\mathfrak{osp}(1|2)$ which are the analogues of discrete series representations, for which explicit tensor product decompositions exist. By diagonalising an explicit operator of $\mathfrak{osp}(1|2)$ in the representations and in the two- and three-fold tensor products we get eigenfunctions of this operator in different ways; the coupled and the uncoupled way. The relations between these eigenvectors leads to the convolution identities, which can also be viewed as an identity for orthogonal polynomials in two variables. The polynomials involved are Bannai–Ito polynomials and super extensions of the Jacobi, Hahn and generalised Hermite polynomials. Using explicit realisations of the representation, we find a bilinear generating function involving Bessel functions.

This is based on joint work with Jean-Michel Lemay and Luc Vinet, both at CRM, U. de Montréal, Canada.

A nonsymmetric version of Okounkov's BC-type interpolation Macdonald polynomials

04.06 **Tom Koornwinder**
(*University of Amsterdam, Netherlands*)
Time: Thursday 25.07., 16:00 - 16:30, Room HS 6

Abstract: In 1998 Okounkov introduced BC-type interpolation Macdonald polynomials. These are symmetric Laurent polynomials which are determined, up to a constant factor, by their vanishing on interpolation points which depend on q and two additional parameters s and t. He also showed that Macdonald-Koornwinder polynomials can be explicitly expanded in terms of products of two such interpolation polynomials, one in the variable and one in the dual variable. This so-called binomial formula specializes in the one-variable case to the usual q-hypergeometric expression for Askey-Wilson polynomials. Furthermore, Okounkov's polynomials allow extra-vanishing, i.e., they vanish not just on the interpolation points, but also on an additional explicit point set.

The talk presents recent work joint with Disveld and Stokman (see arXiv:1808.01221) where we introduce a nonsymmetric version of Okounkov's polynomials. These are Laurent polynomials (no longer symmetric) characterized by their vanishing on interpolation points. The symmetric Okounkov polynomials can be expressed as a sum over the Weyl group for BC_n of the nonsymmetric polynomials. The existence proof of the nonsymmetric polynomials is by a nested induction process. There are experimental indications for extra-vanishing of the nonsymmetric polynomials.

Riesz distributions and the Wallach set in Dunkl theory

04.07 **Margit Rösler**
(*Paderborn University, Germany*)
Time: Thursday 25.07., 16:30 - 17:00, Room HS 6

Abstract: We introduce Riesz distributions associated with rational Dunkl operators of type A, which are closely related to the well-known Riesz distributions on symmetric cones, such as cones of positive-definite matrices. The study of these distributions relies on the rigorous foundation of a suitable Laplace transform in the Dunkl setting, which goes back to Macdonald but had been established so far only on a formal level. In particular, we shall present an analogue of a famous result of Gindikin for symmetric cones, which states that a Riesz distribution is actually a positive measure if and only if its index belongs to the so-called Wallach set. Besides the Laplace transform, Jack polynomial expansions play an important role in the proofs.

Multidimensional matrix inversions and elliptic hypergeometric series on root systems

04.08 **Hjalmar Rosengren**
(*Chalmers University of Technology and University of Gothenburg, Sweden*)
Time: Thursday 25.07., 17:00 - 17:30, Room HS 6

Abstract: Explicit matrix inversions give a powerful tool for studying hypergeometric series in one or several variables. We will discuss some new multidimensional matrix inversions and their applications to elliptic hypergeometric series on root systems. The talk is based on joint work with Michael Schlosser.

Multivariate Chebyshev polynomials in algebraic signal processing

04.09 **Bastian Seifert**
(*Universität Würzburg, Germany*)
Time: Friday 26.07., 10:30 - 11:00, Room HS 6

Abstract: Algebraic signal processing theory is a unified setting for various linear signal processing concepts. In this setting one can derive fast algorithms for the computation of Fourier transforms of suitable signal models based on a decomposition property of polynomials. In this talk we first give a short introduction to algebraic signal processing theory. Then we will explain why the multivariate Chebyshev polynomials associated to root systems are powerful building blocks for signal models. Furthermore we present a geometric interpretation of the fast algorithms corresponding to the multivariate Chebyshev polynomials.

Multivariate Meixner, Charlier and Krawtchouk polynomials

04.10 **Genki Shibukawa**
(*Kobe University, Japan*)
Time: Friday 26.07., 11:00 - 11:30, Room HS 6

Abstract: In a previous paper (Journal of Lie Theory 26 (2016) 439–477), we introduced a multivariate analogue of Meixner, Charlier and Krawtchouk polynomials and established their main properties; generating functions, orthogonality, difference equations (recurrence formulas). Our multivariate Meixner, Charlier and Krawtchouk polynomials are also regarded as 2 (or 1) parameter deformations of rational analogue for Macdonald polynomials of type A. However, our proofs are based on harmonic analysis on symmetric cones (special functions for matrix arguments) and all our results need a restriction condition for the coupling constant. Recently, we give new their proofs without using harmonic analysis on symmetric cones, and succeed in extending all our previous results for any coupling constant. We would like to talk about these recent advances.

Racah problems for the oscillator algebra and $\mathfrak{sl}_n$

04.11 **Wouter van de Vijver**
(*Ghent University, Belgium*)
Time: Friday 26.07., 11:30 - 12:00, Room HS 6

Abstract: We consider the tensor product of n copies of the oscillator algebra $\mathfrak{h}$. Using the Hopf structure and Casimir operator of $\mathfrak{h}$, we construct a subalgebra $\mathcal{R}_n(\mathfrak{h})$ in the same way the higher rank Racah algebra was constructed for $\mathfrak{su}(1,1)$ in [1]. One can embed the algebra $\mathcal{R}_n(\mathfrak{h})$ into $\mathfrak{sl}_{n-1}$ after an affine transformation of the generators by central elements. We study the connection between recoupling coefficients for $\mathfrak{h}$ and $\mathfrak{sl}_n$-representations. These coefficients turn out to be multivariate Krawtchouck polynomials. The relation with the Wigner-$3nj$ symbols for $\mathfrak{h}$ is explained. Flipping two factors in the tensor product is a symmetry of $\mathcal{R}_n(\mathfrak{h})$. This leads to an automorphism of $\mathfrak{sl}_{n-1}$. The corresponding group elements of $\mathrm{SL}(n-1)$ are constructed.

This is joint work with Nicolas Crampé and Luc Vinet.

[1] H. De Bie, V.X. Genest, L. Vinet, W. van de Vijver, A higher rank Racah algebra and the $(\mathbb{Z}_2)^n$ Laplace-Dunkl operator. *J. Phys. A: Math. Theor.* 51 025203 (20pp), 2018.

MS05: Multiple orthogonal polynomials and Hermite-Padé approximation

Organizer: Walter van Assche (*Katholieke Universiteit Leuven, Belgium*)

Multiple orthogonal polynomials are polynomials in one variable that satisfy orthogonality relations with respect to r measures. They appear in a natural way in Hermite-Padé approximation, which is simultaneous rational approximation to r functions near infinity. The session will focus on special families of multiple orthogonal polynomials, asymptotic behavior of the zeros, asymptotic results and Riemann-Hilbert/steepest descent, applications in numerical analysis, random matrices, determinantal point processes.

Asymptotics of Cauchy biorthogonal polynomials

05.01 **Guillermo López Lagomasino**
(*Universidad Carlos III de Madrid, Spain*)
Time: Monday 22.07., 10:30 - 11:00, Room HS 3

Abstract: We consider sequences of biorthogonal polynomials with respect to a Cauchy type convolution kernel and give the weak and ratio asymptotic of the corresponding sequences of biorthogonal polynomials. The construction is intimately related with a mixed type Hermite-Padé approximation problem whose asymptotic properties is also revealed.

This is joint work with Sergio Medina Peralta and Ulises Fidalgo

Asymptotics of multiple orthogonal polynomials for cubic weight

05.02 **Andrei Martínez Finkelshtein**
(*Baylor University, Texas, USA*)
Time: Monday 22.07., 11:00 - 11:30, Room HS 3

Abstract: We consider the type I and type II multiple orthogonal polynomials (MOPs), satisfying non-hermitian orthogonality with respect to the weight $\exp(-z^3)$ on two unbounded contours on the complex plane. Under the assumption that the orthogonality conditions are distributed with a fixed proportion α, we ?nd the detailed (rescaled) asymptotics of these MOPs, and describe the phase transitions of this limit behavior as a function of α. This description is given in terms of the vector critical measure, the saddle point of the energy functional comprising both attracting and repelling forces. These critical measures are characterized by a cubic equation (spectral curve), and their components live on trajectories of a canonical quadratic differential on the Riemann surface of this equation. The structure of these trajectories and their deformations as function of α plays the crucial role.

This is a joint work with Guilherme L. Silva (University of Michigan, Ann Arbor).

Asymptotics of the recurrence coefficients of multiple orthogonal polynomials for Angelesco systems

05.03 **Maxim Yattselev**
(*Indiana University-Purdue University, Indianapolis, USA*)
Time: Monday 22.07., 11:30 - 12:00, Room HS 3

Abstract: In this talk I will describe asymptotics of the multiple orthogonal polynomials and their recurrence coefficients for an Anglesco system of two measures (measures are absolutely continuous with respect to the Lebesgue measure and have non-vanishing smooth densities) along all sequences of indices (n_1, n_2) for which n_1/n_2 has a limit (possibly infinite). Application to the recovery of the essential spectrum of a Jacobi operator on a 2-homogeneous rooted tree will be provided.

Joint work with A. I. Aptekarev and S. A. Denisov

On matrix Cauchy biorthogonal polynomials

05.04 **Sergio Medina Peralta**
(*Universidad Carlos III de Madrid, Spain*)
Time: Monday 22.07., 12:00 - 12:30, Room HS 3

Abstract: In this talk, we study the sequences of matrix Cauchy biorthogonal polynomials. We will focus on the algebraic aspects of the problem, finding a connection with a kind of mixed type Hermite-Padé approximation problem, and with its corresponding Riemann–Hilbert problem. From here we deduce some of the properties of these sequences of biorthogonal polynomials.

Some characterization problems related to d-orthogonal polynomial sets

05.05 **Hamza Chaggara**
(*Sousse University, Tunisia*)
Time: Monday 22.07., 15:30 - 16:00, Room HS 3

Abstract: The notion of d-orthogonal polynomials is a generalization of the notion of orthogonality in the sense that the polynomials $P_n,\ n = 0, 1, \dots$ satisfy orthogonality conditions with respect to d forms. d-orthogonal polynomials are characterized by a higher-order recurrence relation of the form

$$P_{n+1}(x) = (x + \alpha_{n+1})P_n(x) + \sum_{k=0}^{d} \binom{n}{k} \beta_k^{(n+1)} P_{n-k}(x),\ \beta_d^{(n+1)} \neq 0\ n \geq 0.$$

We are interested, in this talk, with some characterization problems for d-orthogonal polynomial sequences when they satisfy additional properties. Indeed

- d-orthogonal polynomials of Sheffer type.
- Symmetric d-orthogonal polynomials of Sheffer type.
- Classical discrete d-orthogonal polynomials.

Matrix Laguerre biorthogonal polynomials via the Riemann–Hilbert Problem

05.06 **Ana Pilar Foulquié Moreno**
(*Universidade de Aveiro, Portugal*)
Time: Monday 22.07., 16:00 - 16:30, Room HS 3

Abstract: We use the Riemann–Hilbert problem, with jump supported on an appropriate curve in the complex plane, for characterizing the matrix biorthogonal polynomials. We apply this characterization for Matrix Laguerre weights and we derive the first and second order differential relations that the fundamental matrix, solution of the Riemann–Hilbert problem satisfies.

This is joint work with Amilcar Branquinho and Manuel Mañas.

On some multiple orthogonal polynomials of a discrete variable

05.07 **Jorge Arvesú Carballo**
(*Universidad Carlos III de Madrid, Spain*)
Time: Monday 22.07., 16:30 - 17:00, Room HS 3

Abstract: This presentation deals with algebraic and analytic properties of some discrete multiple orthogonal polynomials. First, introduce some special families of multiple orthogonal polynomials that are

q-analogous to discrete families given in [2]. Second, the raising and lowering operators, Rodrigues-type formula, and recurrence relations are discussed. Last, a connection with physical model involving weakly integrable systems [3] as well as with the weak asymptotics [1] for the studied families of multiple orthogonal polynomials will be shown.

This is a joint work with A. M. Ramírez-Aberasturis.

[1] A. I. Aptekarev, J. Arvesú, *Asymptotics for multiple Meixner polynomials*, J. Math. Anal. Appl. **411** (2014), 485–505.
[2] J. Arvesú, J. Coussement, W. Van Assche, *Some discrete multiple orthogonal polynomials*, J. Comput. Appl. Math. **153** (2003), 19–45.
[3] H. Miki, S. Tsujimoto, L. Vinet, A. Zhedanov, *An algebraic model for the multiple Meixner polynomials of the first kind*, J. Phys. A: Math. Theor. **45** (2012), 325205 (11 pp).

Multiple orthogonal polynomials associated with confluent hypergeometric functions

05.08

Hélder Lima
(*University of Kent, Canterbury, UK*)
Time: Monday 22.07., 17:00 - 17:30, Room HS 3

Abstract: In this talk we analyse a new family of multiple orthogonal polynomials of the hypergeometric type, whose orthogonality weights can be described using confluent hypergeometric functions (also known as Kummer or Tricomi functions) of the second kind, discuss some of their differential and recursive properties and exhibit an explicit formula for the polynomials. Particular cases are related with threefold symmetric multiple orthogonal polynomials.

This is ongoing work with Ana Loureiro.

Laguerre-Angelesco multiple orthogonal polynomials on an r-star

05.09

Marjolein Leurs
(*Katholieke Universitait Leuven, Belgium*)
Time: Tuesday 23.07., 10:30 - 11:00, Room HS 3

Abstract: The classical orthogonal polynomials consist of the Jacobi, Hermite and Laguerre polynomials. These can be generalized in a number of ways to multiple orthogonal polynomials which are orthogonal with respect to a system of r measures. In this talk we briefly state results of an extension of the Laguerre polynomials, the Laguerre-Angelesco multiple orthogonal polynomials. The Laguerre-Angelesco polynomials are orthogonal with respect to r measures which all have the same weight function $|x|^\beta e^{-x^r}$, each supported on a ray of an r-star. There are two types of multiple orthogonal polynomials. For each type we give explicit expressions for the polynomials, a differential equation and the asymptotic behavior of the zeros of the polynomials.

Multiple orthogonal polynomials living on a star: ratio asymptotics and zero limiting distributions

05.10

Ana Filipa Loureiro
(*University of Kent, Canterbury, UK*)
Time: Tuesday 23.07., 11:00 - 11:30, Room HS 3

Abstract: At the centre of this talk are polynomials satisfying higher order recurrence relations with all recurrence coefficients, except the last one, equal to zero. The polynomials at issue are orthogonal with respect to a vector of measures, are rotational invariant and all the zeros lie on a star in the complex plane. The discussion will focus on their ratio asymptotic behaviour as well as the zero limit distribution. The emphasis is on those polynomials whose recurrence coefficients are unbounded, but such that after scaling

become asymptotic periodic. The study is motivated and will be illustrated by examples of polynomials associated with confluent hypergeometric weight functions.

This is a joint work with Walter Van Assche.

Discrete multiple orthogonal polynomials on shifted lattices

05.11 **Alexander Dyachenko**
(*University College London, UK / Keldysh Institute of Applied Math., Moscow, Russia*)
Time: Tuesday 23.07., 11:30 - 12:00, Room HS 3

Abstract: There are many ways to define multiple orthogonal polynomials with respect to the classical continuous weights. The approach as in [1,2,3] preserves a kind of the Rodrigues formula, which is a very useful property. We focus on adapting this approach for the discrete case — bearing in mind the deep connection between the classical discrete and continuous orthogonality.

The talk is devoted to a new class of polynomials of multiple orthogonality with respect to the product of classical discrete weights on integer lattices with noninteger shifts. We obtain explicit representations in the form of the Rodrigues formulas. The case of two weights will be presented in more detail.

This is joint work with Vladimir Lysov.

[1] A. I. Aptekarev. Multiple orthogonal polynomials. *J. Comput. Appl. Math.* 99 (1998), no. 1–2, 423–447.
[2] A. I. Aptekarev, F. Marcellán, I. A. Rocha. Semiclassical multiple orthogonal polynomials and the properties of Jacobi-Bessel polynomials. *J. Approx. Theory* 90 (1997), no. 1, 117–146.
[3] W. Van Assche, E. Coussement. Some classical multiple orthogonal polynomials. *J. Comput. Appl. Math.* 127 (2001), no. 1–2, 317–347.

On the eigenvalues of Hermitian Brownian motion in critical situations

05.12 **Thorsten Neuschel**
(*Universität Bielefeld, Germany*)
Time: Tuesday 23.07., 12:00 - 12:30, Room HS 3

Abstract: We study the local behavior of the eigenvalues of Hermitian Brownian motion for large dimensions. These random eigenvalues form a determinantal point process for which in non-critical situations it is known that the local correlations show sine-kernel universality in the bulk of the spectrum. In this talk we focus on certain critical situations depending on the behavior of the initial configuration and show that the (multi-time) correlations exhibit Airy- or Pearcy-kernel universality.

The results presented are based on joint work with Tom Claeys and Martin Venker and they are part of a project which is still in progress.

Multiple Askey–Wilson polynomials and related multiple orthogonal polynomials

05.13 **Walter Van Assche**
(*Katholieke Universitait Leuven, Belgium*)
Time: Tuesday 23.07., 15:30 - 16:00, Room HS 3

Abstract: We first show how one can obtain Al-Salam–Chihara polynomials, continuous dual q-Hahn polynomials, and Askey–Wilson polynomials from the little q-Laguerre and the little q-Jacobi polynomials by using special transformations. This procedure is then extended to obtain multiple Askey–Wilson, multiple continuous dual q-Hahn, and multiple Al-Salam–Chihara polynomials from the multiple little q-Laguerre and the multiple little q-Jacobi polynomials.

This is joint work with Jean Paul Nuwacu (Université du Burundi)

Multilateral inversion of hypergeometric series

05.14 **Michael J. Schlosser**
(*University of Vienna, Austria*)
Time: Tuesday 23.07., 16:00 - 16:30, Room HS 3

Abstract: From Gustafson's A_r extension of Dougall's ${}_2H_2$ summation theorem we deduce a new explicit multilateral matrix inverse. As an application, we obtain a summation theorem for a specific A_r ${}_3H_3$ series.

A survey on biorthogonal polynomials and functions

05.15 **Clemente Cesarano**
(*Section of Mathematics – Uninettuno University, Roma, Italy*)
Time: Tuesday 23.07., 16:30 - 17:00, Room HS 3

Abstract: The theory of orthogonal polynomials is well established and detailed, covering a wide field of interesting results, as in particular for solving certain differential equations. On the other side the concepts and the related formalism of the theory of bi-orthogonal polynomials is less developed and much more limited. By starting from the orthogonality properties satisfied from the ordinary and generalized Hermite polynomials is possible to derive a further family (known in literature) of these kind of polynomials which are bi-orthogonal with their adjoint. This aspect allows us to introduce functions recognized as bi-orthogonal and investigate further generalizations of families of orthogonal polynomials. Furthermore, the use of the so called Chebyshev systems allows us to state relevant relations with bi-orthogonal polynomials with positive kernel.

Multiple orthogonal polynomials interpretation of some high order Toda systems

05.16 **Amilcar Branquinho**
(*Universidade de Coimbra, Portugal*)
Time: Tuesday 23.07., 17:00 - 17:30, Room HS 3

Abstract: Some discrete dynamical systems defined by a Lax pair are considered. The method of investigation is based on the analysis of the matrical moments for the main operator of the pair. The solutions of these systems are studied in terms of properties of this operator, giving, under some conditions, explicit expressions for the resolvent function.

MS06: Symbolic computation and special functions

Organizers: Manuel Kauers (*Johannes Kepler University Linz, Austria*)
Veronika Pillwein (*Johannes Kepler University Linz, Austria*)

Computer algebra plays an increasingly important role in the investigation of special functions. For large classes of special functions we now have strong algorithmic theories. Software packages based on these theories successfully solve interesting problems that are not accessible by other means and they also routinely and reliably solve tedious subproblems that frequently arise in day-to-day calculations. The purpose of this minisymposium is to join computer algebra people interested in special functions with special functions people interested in computer algebra, in order to share recent trends, new techniques, and open problems at the intersection of these two areas.

Rigorous numerical evaluation of D-finite functions in SageMath

06.01 **Marc Mezzarobba**
(*Sorbonne Université, Campus Pierre et Marie Curie, Paris, France*)
Time: Tuesday 23.07., 10:30 - 11:00, Room HS 5

Abstract: I will give a demo of the symbolic-numeric features available for working with D-finite functions in the Sage package `ore_algebra`.

Recall that a complex analytic function is called D-finite when it satisfies a linear ODE with polynomial coefficients. D-finite functions form a class analogous to that of hypergeometric functions, but more general. They come up in areas such as analytic combinatorics and mathematical physics, and lend themselves well to symbolic manipulation by computer algebra systems.

At the heart of the analytic features of `ore_algebra` is a rigorous implementation of numerical analytic continuation of D-finite functions. Numerical analytic continuation consists in computing numerical approximation of the transition matrices that maps initial values of an ODE somewhere on the complex plane to initial values elsewhere that define the same solution. The implementation is rigorous in the sense that it returns not just an approximation but an enclosure of the exact mathematical result.

Numerical analytic continuation is the basic brick for computing values of D-finite functions anywhere on their Riemann surfaces, rigorous polynomial approximations of D-finite functions on real or complex domains, monodromy matrices of differential operators, and other related objects. The code fully supports the important limit case where the (generalized) initial values are provided at regular singular points of the ODE, making it possible in particular to compute connection constants between regular singularities.

Implementing finite summation identities of polygamma and related functions into Mathematica

06.02 **Lu Wei**
(*University of Michigan, Dearborn, USA*)
Time: Tuesday 23.07., 11:00 - 11:30, Room HS 5

Abstract: Finite sums of polygamma and related functions find diverse applications in mathematical physics and other fields. These types of sums often admit closed-form expressions by exploring recurrence relations of various forms. In this talk, we will first outline some strategies to approach the considered sums. We will then discuss the on-going collaboration with Wolfram Research to possibly implement these sums into future versions of the computer algebra system Mathematica.

Orthogonal polynomials for higher-order Euler polynomials

06.03 **Lin Jiu**
(*Department of Mathematics and Statistics, Dalhousie University, Halifax, Canada*)
Time: Tuesday 23.07., 11:30 - 12:00, Room HS 5

Abstract: Since recent results recognize higher-order Euler polynomials as the moments of certain random variables, it is natural to study the corresponding monic orthogonal polynomials. Based on the orthogonal polynomials with respect to the Euler numbers, obtained by Carlitz and Al-Salam, we identify the orthogonal polynomials with respect to higher-order Euler polynomials are the Meixner-Pollaczek polynomials, with certain arguments and constant factors. Applications, based on the connection to generalized Motzkin numbers, involve matrix and lattice path representations. Analogues for Bernoulli numbers and Bernoulli polynomials are also presented. This is joint work with Diane Y. H. Shi

Positive systems of polynomial equations

06.04 **Michael Drmota**
(*Technische Universität Wien, Austria*)
Time: Tuesday 23.07., 12:00 - 12:30, Room HS 5

Abstract: A positive system of polynomial equations is of the form $y = P(x, y)$, where $y = (y_1, \ldots, y_k)$ is a k-dimensional vector and $P(x, y) = (P_1(x, y), \ldots, P_k(x, y))$ a system of k polynomials with non-negatives coefficients in x and $y = (y_1, \ldots, y_k)$. Under quite natural conditions such systems have a unique solution $y(x) = (y_1(x), \ldots, y_k(x))$ of power series in x that have - by construction - non-negative coefficients and are algebraic functions. Such positive systems of polynomial equations appear naturally in many combinatorial questions. In contrast to arbitrary algebraic functions the Puiseux expansion at the dominant singularity of these functions (that determines the asymptotic behavior of the coefficients) is quite restricted, in particular the exponent can only be a dyadic rational number. This has been shown by Banderier and Drmota in 2015. Since we are in the framework of algebraic functions it is clear that full asymptotics of the coefficients of the functions $y_j(x)$ can be automatically determined. It is, however, a non-trivial problem to make this computation efficient, in particular for large systems of equations. The purpose of this talk to introduce the main results on positive systems of equations and to pose the efficiency computational question as an open problem.

Polynomials from $(0, m, s)$-nets and Walsh functions

06.05 **Elaine Wong**
(*RICAM, Austrian Academy of Sciences, Linz, Austria*)
Time: Wednesday 24.07., 10:30 - 11:00, Room HS 5

Abstract: We consider the problem of integrating a function f over the unit hypercube of dimension s. In practice, a digital net (a discrete breakdown of the continuous interval) can be cast over the unit hypercube in a way such that performing the integration over this net effectively and accurately estimates the integral. In our present work, we consider the integration of the joint probability density function of distinct points randomly chosen from a scrambled $(0, m, s)$-net and multivariate Walsh functions. This idea expands on the work of Wiart and Lemieux (2019). It allows us to simplify the integral into a discrete, symbolic sum containing the parameters m and s. From there, we are able to construct a certain univariate polynomial which can be used to determine how well the integration performs compared to the more widely used Monte-Carlo and other equidistribution methods. The coefficients of this polynomial conveniently contain hypergeometric series in the parameters. In this talk, we illustrate how to use available symbolic computation machinery to simplify such a polynomial into a nice closed form consisting of beta functions, from which we can draw our desired conclusions.

Computer algebra for basic hypergeometric functions

06.06 **Christoph Koutschan**
(*RICAM, Austrian Academy of Sciences, Linz, Austria*)
Time: Wednesday 24.07., 11:00 - 11:30, Room HS 5

Abstract: With the exception of q-hypergeometric summation, the use of computer algebra packages implementing Zeilberger's holonomic systems approach in a broader mathematical sense is less common in the field of q-series and basic hypergeometric functions. As a case study, we look at the celebrated Ismail-Zhang formula, an important q-analog of a classical expansion formula of plane waves in terms of Gegenbauer polynomials, and demonstrate how the Mathematica package HolonomicFunctions can be employed to generate a computer-assisted proof of this identity. The HolonomicFunctions package was originally developed for dealing with classical special function identities (sums, series, integrals), but its range of applicability also includes q-series and q-orthogonal polynomials. This is joint work with Peter Paule.

An algorithmic summation theory for indefinite nested sums and products

06.07 **Carsten Schneider**
(*Research Institute for Symbolic Computation, Johannes Kepler University, Linz, Austria*)
Time: Wednesday 24.07., 11:30 - 12:00, Room HS 5

Abstract: Inspired by Karr's pioneering work (1981) we developed a summation theory of difference rings that enables one to rephrase special functions in terms of indefinite nested sums in the setting difference rings. Within this representation certain optimality criteria are fulfilled: e.g., the objects represented in the difference ring (except elements such as the alternating sign) do not satisfy any polynomial relations or the nesting depth of the arising sums have minimal nesting depth. Combining such optimal representations of special functions in combination with definite summation algorithms, like creative telescoping and recurrence solving in the setting of difference rings, yield a strong summation machinery for practical problem solving. We will demonstrate these features implemented in the summation package Sigma by concrete examples coming from combinatorics and particle physics.

The mathematical functions grimoire

06.08 **Fredrik Johansson**
(*INRIA Bordeaux and Institut de Mathématiques de Bordeaux, France*)
Time: Wednesday 24.07., 12:00 - 12:30, Room HS 5

Abstract: Fungrim (http://fungrim.org) is a new, open source database of formulas and tables for special functions. All formulas are encoded symbolically and include explicit conditions of validity for the variables, representing rewrite rules that can be applied mechanically without implicit exceptions (for instance, regarding branch cuts).

The immediate goal is to create a web-based special functions reference work that addresses some of the drawbacks of resources such as the NIST Digital Library of Mathematical Functions, the Wolfram Functions site, and Wikipedia. A potential longer-term ambition is to provide a software library for symbolic knowledge about special functions, usable by computer algebra systems and theorem proving software.

This talk will discuss the motivation behind the project, design issues, and possible applications. The project is still in experimental stages, and feedback is invited.

Interpolated sequences and critical L-values of modular forms

06.09 **Armin Straub**
(*University of South Alabama, USA*)
Time: Thursday 25.07., 10:30 - 11:00, Room HS 5

Abstract: It is well-known that the Apéry numbers which arise in the irrationality proof for $\zeta(3)$ satisfy many interesting arithmetic properties and are related to the Fourier coefficients of a weight 4 modular form. Recently, Zagier expressed an interpolated version of these numbers in terms of a critical L-value of the same modular form. We discuss this evaluation as well as extensions, including to interpolations of Zagier's six sporadic sequences. Our focus is on applications of and challenges for computer algebra that come up naturally in the context of these evaluations. This talk is based on joint work with Robert Osburn.

Efficient rational creative telescoping

06.10 **Hui Huang**
(*University of Waterloo, Canada*)
Time: Thursday 25.07., 11:00 - 11:30, Room HS 5

Abstract: Since 1990s, creative telescoping has become the cornerstone for evaluating definite sums of discrete special functions in computer algebra. Various algorithmic generalizations and improvements for this technique have been developed over the past two decades. At the present time, the reduction-based approach has gained the most support as it is both efficient in practice and has the important feature of being flexible to find a telescoper for a given function with or without construction of a certificate. There is, however, one handicap of this approach. That is, the approach can suffer from intermediate expression swell, especially in the part of a certificate, even if the final output ends up to be small.

In this talk, we present a new algorithm to compute minimal telescopers for rational functions in two discrete variables. This is the first step towards the long-term goal of developing fast creative telescoping algorithms for special functions that circumvent intermediate expression swell. As with the reduction-based approach, our algorithm also has the nice feature that the computation of a telescoper is independent of its certificate. Moreover, our algorithm uses a sparse representation of the certificate, which allows to be more easily manipulated and analyzed without knowing the precise expanded form. This sparse representation hides any potential exponential expression swell until the final (and optional) expansion. A complexity analysis, along with a Maple implementation, suggests that our algorithm has better theoretical and practical performances than the reduction-based approach when restricted to the rational case. This is joint work with M. Giesbrecht, G. Labahn and E. Zima.

Inverse Zeilberger's Problem

06.11 **Marko Petkovšek**
(*Faculty of Mathematics and Physics, University of Ljubljana, Slovenia*)
Time: Thursday 25.07., 11:30 - 12:00, Room HS 5

Abstract: Given a proper hypergeometric term $F(n,k)$, Zeilberger's *Creative Telescoping algorithm* finds a linear recurrence with rational coefficients satisfied by the sequence $s_n = \sum_{k=0}^{n} F(n,k)$. In the context of solving recurrence equations, we consider here what might be called the *inverse Zeilberger's problem*: given a homogeneous linear recurrence with polynomial coefficients, find its solutions representable as definite sums of a certain form.

As a first step in this direction, we provide an algorithm which, given a linear recurrence operator L with polynomial coefficients, and a product of binomial coefficients of the form

$$F(n,k) \;=\; \prod_{i=1}^{m} \binom{a_i n + b_i}{k}$$

where a_i are positive integers and b_i are arbitrary constants, returns a linear recurrence operator L' with rational coefficients such that for any sequence y of the form $y_n = \sum_{k=0}^{\infty} F(n,k)h_k$, we have $Ly = 0$ if and only if $L'h = 0$. This enables us to find all such solutions y where h belongs to a class of holonomic sequences with a known algorithm for converting from recursive to explicit representation.

Sparse polynomial interpolation with arbitrary orthogonal polynomial bases

06.12

Erdal Imamoglu
(*Department of Mathematics, North Carolina State University, USA*)
Time: Thursday 25.07., 12:00 - 12:30, Room HS 5

Abstract: An algorithm for interpolating a polynomial f from evaluation points whose running time depends on the sparsity t of the polynomial when it is represented as a sum of t Chebyshev polynomials of the first kind with non-zero scalar coefficients is given by Lakshman and Saunders [SIAM J. Comput., vol. 24, nr. 2 (1995)]; Kaltofen and Lee [JSC, vol. 36, nr. 3–4 (2003)] analyze a randomized early termination version which computes the sparsity t. Those algorithms mirror Prony's algorithm for the standard power basis to the Chebyshev basis of the first kind. An alternate algorithm by Arnold's and Kaltofen's [Proc. ISSAC 2015, Sec. 4] uses Prony's original algorithm for standard power terms. Here we give sparse interpolation algorithms for generalized Chebyshev polynomials, which include the Chebyshev bases of the second, third and fourth kind. Our algorithms also reduce to Prony's algorithm. If given on input an upper bound B for the sparsity t, our new algorithms deterministically recover the sparse representation in the first, second, third and fourth kind Chebyshev representation from exactly $t + B$ evaluations. Finally, we generalize our algorithms to bases whose Chebyshev recurrences have parametric scalars. We also show how to compute those parameter values which optimize the sparsity of the representation in the corresponding basis, similar to computing a sparsest shift.

This is a joint work with Erich L. Kaltofen (North Carolina State University and Duke University) and Zhengfe.

MS07: Recent trends in asymptotics

Organizer: Gergő Nemes (*Alfréd Rényi Institute of Mathematics, Budapest, Hungary*)

The main goal of the mini-symposium is to bring together researchers from all over the world to discuss their most recent results in the field of asymptotic analysis. The mini-symposium is intended to cover a broad range of subjects in asymptotic analysis, including classical asymptotics, uniform asymptotics, hyperasymptotics, resurgent function theory and exact WKB analysis.

From asymptotics to exact results: unraveling the analytic structure of solutions of Painlevé I

07.01

Inês Aniceto
(*University of Southampton, UK*)
Time: Monday 22.07., 10:30 - 11:00, Room HS 5

Abstract: Understanding the asymptotic properties of solutions of the Painlevé I non-linear ODE is of great interest in both mathematics and physics. It is well known that the asymptotic behaviour of these solutions is connected to the existence of exponentially small contributions, directly linked to physical phenomena not captured by a perturbative analysis. The theory of resurgence perfectly captures this perturbative/non-perturbative connection and its consequences. Moreover, it allows us to construct a full non-perturbative solution from perturbative data. In this talk, I will demonstrate the essential role of resurgence theory, coupled to exponentially accurate numerical methods, in going beyond the perturbative results and obtain (analytically and numerically) non-perturbative data. In particular, I will exemplify how these techniques can be applied to to the calculation of poles of Painlevé I solutions.

Invisible catastrophes: when to turn an asymptotic blind eye

07.02

Christopher J. Howls
(*University of Southampton, UK*)
Time: Monday 22.07., 11:00 - 11:30, Room HS 5

Abstract: Recent work on high-frequency flow-interaction effects for 3D jet engine noise in a moving media uncovered unusual structures that appeared to violate mathematical aspects of both classical ray analysis and catastrophe theory whereby caustics appeared to end at finite ordinary (as opposed to turning) points in real space. A careful study of the local (complex) ray structure led to the introduction of a novel set of special functions that, a posteriori, resolved these problems. These special functions possess interesting novel properties from the point of view of analysis, with implications for exponential asymptotics. These special functions are now seen to occur in a variety of physical situations, ranging from the original aeroacoustic problem, to black hole event horizons and nonlinear traveling waves such as the Severn Bore. In this talk we will introduce these special functions and give an introduction to their novel properties.

Gaussian unitary ensembles with pole singularities near the soft edge and a system of coupled Painlevé XXXIV equations

07.03

Dan Dai
(*City University of Hong Kong*)
Time: Monday 22.07., 11:30 - 12:00, Room HS 5

Abstract: In this paper, we study the singularly perturbed Gaussian unitary ensembles defined by the measure

$$\frac{1}{C_n} e^{-n \operatorname{tr} V(M;\lambda,\vec{t})} dM,$$

over the space of $n \times n$ Hermitian matrices M, where $V(x; \lambda, \vec{t}\,) := 2x^2 + \sum_{k=1}^{2m} t_k (x - \lambda)^{-k}$ with $\vec{t} = (t_1, t_2, \dots, t_{2m}) \in \mathbb{R}^{2m-1} \times (0, \infty)$, in the multiple scaling limit where $\lambda \to 1$ together with $\vec{t} \to \vec{0}$ as $n \to \infty$ at appropriate related rates. We obtain the asymptotics of the partition function, which is described explicitly in terms of an integral involving a smooth solution to a new coupled Painlevé system generalizing the Painlevé XXXIV equation. The large n limit of the correlation kernel is also derived, which leads to a new universal class built out of the Ψ-function associated with the coupled Painlevé system.

This is a joint work with Shuai-Xia Xu and Lun Zhang.

Distribution of the maximal height of N non-intersecting Bessel paths

07.04 **Luming Yao**
(*City University of Hong Kong*)
Time: Monday 22.07., 12:00 - 12:30, Room HS 5

Abstract: Consider N non-intersecting Bessel paths start at a positive position $x = a > 0$ when time $t = 0$ and end at the origin $x = 0$ when time $t = 1$. Using the Karlin–McGregor formula and the Schur function expansion of the corresponding determinants, we derive the exact distribution function for the maximal height of the outermost path, which is given in terms of Hankel determinants of the multiple orthogonal polynomials.

Voros coefficients and the topological recursion for the hypergeometric differential equations associated with the 2-dimensional Garnier system

07.05 **Yumiko Takei**
(*Department of Mathematics, Graduate School of Science, Kobe University, Japan*)
Time: Monday 22.07., 15:30 - 16:00, Room HS 5

Abstract: The N-dimensional Garnier system is a Hamiltonian system with N independent variables obtained through monodromy preserving deformations of second order linear differential equations on $\mathbb{P}^1$ with $N+3$ regular singular points. In the case of $N = 1$, the system reduces to the sixth Painlevé equation P_{VI} and the Gauss hypergeometric function gives a particular solution of P_{VI}. Note that each member of the family of the Gauss hypergeometric equations is obtained from P_{VI} by the so-called confluence process. In the same manner the hypergeometric differential equations with two independent variables are obtained from the 2-dimensional Garnier system. In this talk I consider some relationship between the exact WKB analysis and the topological recursion for the hypergeometric differential equations with two independent variables.

Exact WKB analysis is a powerful tool to study differential equations globally. In particular, Voros coefficients provide important quantities for describing global behavior of solutions of differential equations. On the other hand, the topological recursion introduced by B. Eynard and N. Orantin [2] to study the correlation functions in the random matrix theory gives a generalization of the loop equations for the matrix model. Recently, several surprising connections between exact WKB analysis and topological recursion have been discovered. For example, it is shown that WKB solutions are constructed via the topological recursion [1]. Furthermore, together with Iwaki and Koike I show that in the case of the family of the Gauss hypergeometric equations Voros coefficients are described by the generating functions of free energies defined in terms of the topological recursion [3,4].

In this talk, I would like to discuss a generalization of the above result, that is, I will report that the Voros coefficients for some confluent hypergeometric differential equations of two variables associated with degenerate 2-dimensional Garnier systems are described by the generating functions of free energies defined in terms of the topological recursion. As its application I will also show that the following objects can be computed in an explicit manner: (i) three-term difference equations that the generating function of the free energies satisfies, (ii) explicit form of free energies, and (iii) explicit form of Voros coefficients.

[1] V. Bouchard and B. Eyanard, Reconstructing WKB from topological recursion, *Journal de l'Ecole polytechnique – Mathematiques*, **4** (2017), pp. 845–908.
[2] B. Eynard and N. Orantin, Invariants of algebraic curves and topological expansion, *Communications in Number Theory and Physics*, **1** (2007), pp. 347–452; arXiv:math-ph/0702045.
[3] K. Iwaki, T. Koike, and Y.-M. Takei, Voros coefficients for the hypergeometric differential equations and Eynard–Orantin's topological recursion, part I: for the Weber equation; arXiv:1805.10945.
[4] K. Iwaki, T. Koike, and Y.-M. Takei, Voros coefficients for the hypergeometric differential equations and Eynard–Orantin's topological recursion, part II: for the confluent family of hypergeometric equations, preprint; arXiv:1810.02946.

The hypergeometric function and WKB solutions

07.06 **Takashi Aoki**
(*Kindai University, Japan*)
Time: Monday 22.07., 16:00 - 16:30, Room HS 5

Abstract: We introduce a large parameter in the three parameters of the hypergeometric differential equation and consider exact WKB solutions. These formal solutions are Borel summable with respect to the parameter under suitable conditions. Taking the Borel sum, we obtain analytic solutions of the equation. We give the relations between the hypergeometric function and Borel resummed WKB solutions. As an application, we have asymptotic expansion formulas of the hypergeometric function with respect to the large parameter.

The confluent hypergeometric function and WKB solutions

07.07 **Toshinori Takahashi**
(*Kindai University, Japan*)
Time: Monday 22.07., 16:30 - 17:00, Room HS 5

Abstract: In this talk, we consider the relation between the confluent hypergeometric function and the Borel resummed WKB solutions. We show that the relations and the Voros coefficients and their Borel sums can be obtained by applying the operation of confluent to those in the case of the hypergeometric equation. We also talk about further confluence.

On functions K and E generated by a sequence of moments

07.08 **Avner Kiro**
(*Tel Aviv University, Israel*)
Time: Tuesday 23.07., 15:30 - 16:00, Room HS 5

Abstract: For a class of functions γ analytic in the sector $\{s\colon |\arg(s)| < \alpha_0\}$ with $\frac{\pi}{2} < \alpha_0 < \pi$, we describe the asymptotic behavior of the analytic function

$$K(z) = \frac{1}{2\pi \mathrm{i}} \int_{c-\mathrm{i}\infty}^{c+\mathrm{i}\infty} z^{-s}\gamma(s)\,\mathrm{d}s,$$

that solves the moment problem

$$\int_0^\infty t^n K(t)\,\mathrm{d}t = \gamma(n+1), \qquad n \geq 0\,,$$

and of the entire function

$$E(z) = \sum_{n\geq 0} \frac{z^n}{\gamma(n+1)}.$$

These two functions naturally appear in various classical problems of analysis. The talk is based on a joint work with M. Sodin.

Transition region expansions

07.09 **Adri B. Olde Daalhuis**
(*University of Edinburgh, UK*)
Time: Tuesday 23.07., 16:00 - 16:30, Room HS 5

Abstract: For the normalised incomplete gamma function $Q(a,z) = \Gamma(a,z)/\Gamma(a)$ we will construct what we call *transition region expansions*, and provide full details of the inversion of these new expansions. These are expansions that are valid in the regions in which $Q(a,z)$ changes dramatically, and their coefficients are polynomials satisfying simple recurrence relations. The region of validity overlaps with those of the non-uniform "outer" expansions. Furthermore, the coefficients of their inversions are simple polynomials, whose computation and implementation are straightforward.

It is surprising to us that these transition region expansions for the normalised incomplete gamma function have not yet been discussed in the literature, given the fact that expansions of similar type for Bessel functions are well known (see §10.19.iii in the DLMF). What our new expansions and the transition region expansions for the Bessel functions have in common is that both mimic the corresponding uniform expansions.

This is a joint work with Gergő Nemes.

On the Borel summability of WKB solutions near a simple pole

07.10 **Gergő Nemes**
(*Alfréd Rényi Institute of Mathematics, Budapest, Hungary*)
Time: Tuesday 23.07., 16:30 - 17:00, Room HS 5

Abstract: We consider the following Schrödinger-type differential equation:

$$\frac{\mathrm{d}^2 W(u,\xi)}{\mathrm{d}\xi^2} = (u^2 + \psi(\xi))W(u,\xi),$$

where u is a large positive parameter and $\psi(\xi)$ is an analytic function of ξ apart from countably many singularities. It is known that this equation has formal solutions of the form

$$W_{1,2}(u,\xi) = \mathrm{e}^{\pm\xi u}\sum_{n=0}^{\infty}\frac{\mathsf{A}_n(\pm\xi)}{u^n}.$$

These are called the WKB solutions. We study the Borel summability of these WKB solutions near a simple pole of the function $\psi(\xi)$ which we assume to be located at the origin. It is shown that both of the formal series are Borel summable in every closed strip $\{\xi : |\Im\xi| \le \gamma\}$ contained in the domain of analyticity of $\psi(\xi)$ apart from the Stokes rays $\arg\xi = 0,\, 2\pi$ and $\arg\xi = \pm\pi$ emanating from the origin. We determine the type of singularities of the Borel transforms near the origin when $|\xi|$ is small and also provide global connection formulae between the solutions $W_1(u,\xi)$ and $W_2(u,\xi)$.

MS08: Asymptotics via non-standard orthogonality

Organizers: Andrei Martínez Finkelshtein (*Baylor University, Texas, USA / Univ. de Almería, Spain*)
Guilherme Silva (*University of Michigan, USA*)
Maxim Yattselev (*Indiana University IUPUI, USA*)

Asymptotic behavior of sequences of polynomials can certainly be called a classical problem. In particular, analytic properties of classical orthogonal polynomials have attracted interest from late 19th century. Nevertheless, in the last few decades the field has experienced several striking developments. On one hand they were stimulated by applications: for instance, many models of mathematical physics can be described in terms of sequences of polynomials exhibiting non-standard type of orthogonality (multiple, non-hermitian, Sobolev, matrix, to mention a few), and their asymptotic analysis is the key to the study of large-scale phenomena. On the other hand, new methods from potential theory, spectral theory and integrable systems have been successfully developed. For this mini-symposium, we plan on bringing together a range of experts in the asymptotic theory of orthogonal polynomials and their generalizations, representing a wide scope of techniques and applications from a modern perspective.

Non-hermitian orthogonality in random tiling problems

08.01 **Arno Kuijlaars**
(*Katholieke Universitait Leuven, Belgium*)
Time: Wednesday 24.07., 10:30 - 11:00, Room HS 3

Abstract: I will discuss how polynomials with a non-hermitian orthogonality on a contour in the complex plane arise in certain random tiling problems. In the case of periodic weightings the setup generalizes to matrix valued orthogonality.

In work with Maurice Duits (KTH Stockholm) the Riemann-Hilbert problem for matrix valued orthogonal polynomials was used to obtain asymptotics for domino tilings of the two-periodic Aztec diamond. This model is remarkable since it gives rise to a gas phase, in addition to the more common solid and liquid phases.

[1] M. Duits and A. B. J. Kuijlaars. The two periodic Aztec diamond and matrix valued orthogonal polynomials. *Preprint arXiv:1712.05636, to appear in J. Eur. Math. Soc.*

Two applications of non-standard orthogonality

08.02 **Alfredo Deaño**
(*University of Kent, UK*)
Time: Wednesday 24.07., 11:00 - 11:30, Room HS 3

Abstract: In this talk we will illustrate the use of non-standard orthogonal polynomials (OPs), more precisely non-Hermitian OPs in the complex plane, and their large degree asymptotics in two different but related contexts: asymptotic behavior of special function solutions of Painlevé differential equations and analysis of non-Hermitian random matrix models with singularities. *Partly based on joint work with Nick Simm (University of Sussex, UK).*

Construction of the global parametrix for the kissing polynomials

08.03 **Andrew Celsus**
(*University of Cambridge, UK*)
Time: Wednesday 24.07., 11:30 - 12:00, Room HS 3

Abstract: When trying to implement the Deift-Zhou method of steepest descent to recover asymptotics of orthogonal polynomials, one needs to construct solutions to a model Riemann-Hilbert problem (RHP).

When studying a certain family of orthogonal polynomials with complex weights known as the kissing polynomials, the model problem does not possess the same symmetries that one usually encounters when dealing with positive weight functions. As such, the construction of the global parametrix, which is the solution of this model problem, requires a different approach. The goal of this talk is to outline the construction of global parametrix which arises when one is trying to study asymptotics of the kissing polynomials. *Joint work with Guilherme Silva of the University of Michigan.*

Generalized Jacobi polynomials on a cross

08.04 **Ahmad Barhoumi**
(*Indiana University IUPUI, USA*)
Time: Wednesday 24.07., 12:00 - 12:30, Room HS 3

Abstract: Polynomials satisfying a non-Hermitian orthogonality relation appear naturally in many places, one of which is the construction of Padé approximants. One feature that sets these polynomials apart from orthogonal polynomials on the real line is that the degree of the polynomial orthogonal up to order n may be less than n. In this talk, I will discuss the asymptotic analysis of a specific family of Jacobi-type polynomials via Riemann–Hilbert Problem while highlighting how the degeneration of degree displays itself in the analysis. *Joint work with Maxim Yattselev.*

Some results and conjectures on the asymptotic properties of polynomials orthogonal over bounded domains

08.05 **Erwin Miña Díaz**
(*University of Mississippi, USA*)
Time: Thursday 25.07., 10:30 - 11:00, Room HS 3

Abstract: In this talk, we will discuss some of the recent progress made in understanding the asymptotic behavior of polynomials orthogonal with respect to the area measure over a bounded domain. We will conjecture what that behavior is in the attractive but elusive case of orthogonality over a domain bounded by a piecewise analytic Jordan curve.

Converting planar orthogonality to orthogonality on a contour

08.06 **Alan Groot**
(*Katholieke Universitait Leuven, Belgium*)
Time: Thursday 25.07., 11:00 - 11:30, Room HS 3

Abstract: We look at a model related to the spherical ensemble, by introducing two points on the sphere that repel particles. The corresponding average characteristic polynomial satisfies a Hermitian orthogonality on the complex plane. We show how to convert this planar orthogonality to non-Hermitian orthogonality with respect to a weight on a collection of contours. *Joint work with Juan Criado del Rey and Arno Kuijlaars.*

Zeros of Faber polynomials for Joukowski airfoils

08.07 **Franck Wielonsky**
(*Centre de Mathématiques et Informatique, Université Aix-Marseille, France*)
Time: Thursday 25.07., 11:30 - 12:00, Room HS 3

Abstract: Let K be the closure of a bounded region in the complex plane with simply connected complement whose boundary is a piecewise analytic curve with at least one outward cusp. The asymptotics of zeros of Faber polynomials for K are not understood in this general setting. Joukowski airfoils provide

a particular class of such sets. We determine the (unique) weak-* limit of the full sequence of normalized counting measures of the Faber polynomials for Joukowski airfoils. This limit is always different from the equilibrium measure of K. This implies that these airfoils admit an electrostatic skeleton and also explains an interesting class of examples of Ullman related to Chebyshev quadrature. *Joint work with Norm Levenberg.*

Detecting outliers with Christoffel-Darboux kernels

08.08 **Bernhard Beckermann**
(*Laboratoire de Mathématiques Paul Painlevé, Université de Lille, France*)
Time: Thursday 25.07., 12:00 - 12:30, Room HS 3

Abstract: Two central objects in constructive approximation, the Christoffel-Darboux kernel and the Christoffel function, are encoding ample information about the associated moment data and ultimately about the possible generating measures. We develop a multivariate theory of the Christoffel-Darboux kernel in $\mathbb{C}^d$, with emphasis on the perturbation of Christoffel functions and their level sets with respect to perturbations of small norm or low rank. The statistical notion of leverage score provides a quantitative criterion for the detection of outliers in large data. Using the refined theory of Bergman orthogonal polynomials, we illustrate the main results, including some numerical simulations, in the case of finite atomic perturbations of area measure of a 2D region. Methods of function theory of a complex variable and (pluri)potential theory are widely used in the derivation of our perturbation formulas. *Joint work with Mihai Putinar (University of California at Santa Barbara), Edward B. Saff (Vanderbilt University) and Nikos Stylianopoulos (University of Cyprus).*

Nikishin systems on star-like sets: limiting functions in ratio asymptotics

08.09 **Abey López García**
(*University of Central Florida, USA*)
Time: Thursday 25.07., 15:30 - 16:00, Room HS 3

Abstract: Let $p \geq 2$ be an integer, and let $\{Q_n\}_{n\geq 0}$ be the sequence of monic, type II multiorthogonal polynomials associated with a Nikishin system of measures supported on a compact subset of the $(p+1)$-star $S_+ = \{z \in \mathbb{C} : z^{p+1} \geq 0\}$. Under some conditions, the sequence of ratios Q_{n+1}/Q_n has periodic limits outside the support of the measures in the Nikishin system. In this talk I will describe the limiting functions in terms of certain conformal algebraic functions defined on a compact Riemann surface of genus zero. *Joint work with G. López Lagomasino.*

On the irrationality and the measure of irrationality of $\log(1 + 1/m)\log(1 - 1/m)$

08.10 **Vladimir Lysov**
(*Keldysh Institute of Applied Mathematics, Russian Academy of Sciences, Moscow, Russia*)
Time: Thursday 25.07., 16:00 - 16:30, Room HS 3

Abstract: We consider the Diophantine approximants for the product of the two logarithms $\gamma_m := \log\left(1+\frac{1}{m}\right)\log\left(1-\frac{1}{m}\right)$ for an integer m. We prove that for all $m \geq 33$ the number γ_m is irrational. This is an improvement of the previous result by M. Hata [1]. We also find new upper estimates of the measure of irrationality of γ_m.

Our approach is based on the Hermite–Padé approximants for the vector of functions (f_1, f_2, f_3), where

$$f_1(z) := \log\left(1+\frac{1}{z}\right), \quad f_2(z) := \log\left(1-\frac{1}{z}\right), \quad f_3 := f_1 f_2.$$

This vector is an example of the Generalized Nikishin system of Markov functions on graphs [2]. The common denominator of the approximants satisfies certain multiple orthogonality relations. The key ingredient

of our proof is an explicit formula for the common denominator. By means of this formula we obtain the asymptotics of the sequence of the approximants and also some remarkable arithmetic properties of them.

[1] M. Hata. The irrationality of $\log(1+1/q)\log(1-1/q)$. *Trans. Amer. Math. Soc.* **350**:6 (1998), 2311–2327.

[2] A. I. Aptekarev, V. G. Lysov. Systems of Markov functions generated by graphs and the asymptotics of their Hermite-Padé approximants. *Mat. Sb.* **201**:2 (2010), 183–234.

Properties of some classes of quasi-orthogonal polynomials

08.11 **Kerstin Jordaan**
(*University of South Africa, Pretoria, South Africa*)
Time: Thursday 25.07., 16:30 - 17:00, Room HS 3

Abstract: In this talk I will prove the quasi-orthogonality of some classes of hypergeometric and q-hypergeometric polynomials that do not appear in the Askey or q-Askey scheme for orthogonal polynomials. The polynomials considered include, as special cases, two ${}_3F_2$ polynomials considered by Dickinson [Proc. Amer. Math. Soc., 12 (1961), 185–194] and a ${}_2F_2$ polynomial with only positive zeros. I will derive three-term recurrence relations and second order differential equations for the quasi-orthogonal polynomials and investigate whether the recurrence coefficients satisfy conditions necessary for orthogonality established recently by Ismail and Wang [J. Math. Anal. Appl., 474(2) (2019), 1178–1197]. The location and interlacing of the real zeros of the polynomials under consideration will also be discussed.

Blaschke Products, numerical ranges, and the zeros of orthogonal polynomials

08.12 **Brian Simanek**
(*Baylor University, Texas, USA*)
Time: Thursday 25.07., 17:00 - 17:30, Room HS 3

Abstract: Our main object of interest will be the location of zeros of orthogonal polynomials on the unit circle. We will discuss some recent developments that relate these zeros to Blaschke products, numerical ranges of matrices, algebraic curves, quadrature measures, and Poncelet's Theorem. Some open problems will be mentioned along the way.

Asymptotics of orthogonal polynomials with unbounded recurrence coefficients

08.13 **Grzegorz Świderski**
(*Mathematical Institute, University of Wrocław, Poland*)
Time: Friday 26.07., 10:30 - 11:00, Room HS 3

Abstract: Let μ be a probability measure on the real line with all moments finite. Let $(p_n : n \geq 0)$ be the corresponding sequence of orthonormal polynomials. It satisfies

$$\begin{gathered} p_0(x) = 1, \quad p_1(x) = \frac{x - b_0}{a_0}, \\ a_{n-1}p_{n-1}(x) + b_n p_n(x) + a_n p_{n+1}(x) = x p_n(x) \quad (n \geq 1), \end{gathered} \tag{1}$$

for some sequences $a_n > 0$ and $b_n \in \mathbb{R}$. Conversely, Favard's theorem states that every sequence of polynomials satisfying (1) is orthonormal with respect to some measure μ. The measure μ is unique if the Carleman condition is satisfied, i.e. when

$$\sum_{k=0}^{\infty} \frac{1}{a_k} = \infty.$$

In the proposed talk we are interested in the asymptotic behaviour of orthogonal polynomials in terms of its recurrence coefficients. Let $N \geq 1$ be an integer. In the analysis the crucial role is played by the so-called N-step transfer matrix defined by

$$X_n(x) = \prod_{j=n}^{n+N-1} \begin{pmatrix} 0 & 1 \\ -\frac{a_{j-1}}{a_j} & \frac{x-b_j}{a_j} \end{pmatrix}.$$

We are going to present the following

Theorem 1. *Let $N \geq 1$ be a positive integer and let $i \in \{0, 1, \ldots, N-1\}$. Suppose that the sequence $(X_{nN+i} : n \in \mathbb{N})$ is of bounded variation and let $\mathcal{X}_i$ be its limit. Let*

$$\Lambda = \{x \in \mathbb{R} : |\operatorname{tr} \mathcal{X}_i(x)| < 2\}.$$

If $\det \mathcal{X}_i = 1$ and the Carleman condition is satisfied, then the measure μ is purely absolutely continuous on Λ and there is a continuous real-valued function η such that

$$\sqrt{a_{kN+i-1}}\, p_{kN+i}(x) = \sqrt{\frac{2|[\mathcal{X}_i(x)]_{21}|}{\pi\mu'(x)\sqrt{4-(\operatorname{tr}\mathcal{X}_i(x))^2}}} \sin\Big(\sum_{j=1}^{k} \theta_j(x) + \eta(x)\Big) + \epsilon_k(x), \qquad x \in \Lambda$$

for explicit θ_j and a constructive upper bound on ϵ_k.

Theorem 1 is an extension of results obtained in Máté-Nevai-Totik (1985), Geronimo-Van Assche (1991) and Aptekarev-Geronimo (2016). Our approach is based on uniform diagonalisation of transfer matrices.

We are going to present the applications of Theorem 1 with unbounded a_n to the asymptotics of Christoffel functions with the rate of convergence and to universality limits of Christoffel-Darboux kernel.

This is a joint work with Bartosz Trojan (Polish Academy of Sciences).

Biorthogonal rational functions involving two parameters and the Christoffel type transformation

08.14

Swaminathan Anbhu
(*Indian Institute of Technology, Roorkee, Uttarakhand, India*)
Time: Friday 26.07., 11:00 - 11:30, Room HS 3

Abstract: In this work, a general T-fraction based on a polynomial map is considered. Two generalized linear matrix pencils of the form $\mathcal{G} - z\mathcal{H}$, where $\mathcal{G}$ and $\mathcal{H}$ are tridiagonal matrices, associated to this polynomial map are considered and the orthogonality of the related Laurent polynomials are discussed. These matrix pencils are useful in constructing two sequences of biorthogonal rational functions, $\{p_n^L(z)\}_{n=0}^{\infty}$ and $\{p_n^R(z)\}_{n=0}^{\infty}$, associated with the parameters a_n and b_n respectively, that form the components of the left and right eigenvectors of the matrix pencil. The procedure for constructing these two families is different from the one given in [3]. These two different sequences of orthogonal rational functions lead to the recurrence relations given by

$$\mathcal{P}_{n+1}(z) = \rho_n(z-\nu_n)\mathcal{P}_n(z) + \tau_n(z-a_n)(z-b_n)\mathcal{P}_{n-1}(z), n \geq 1,$$

with initial conditions $\mathcal{P}_0(z) = 1$ and $\mathcal{P}_1(z) = \rho_0(z-\nu_0)$ that are defined on the unit circle as well in the real line. These are known as R_{II} type recurrence relations and were studied by Ismail and Masson [2] and Zhedanov [4] independently. A particular case is considered that provides a Christoffel type transformation of the generalized eigenvalue problem with a reformulation different from the existing literature. Specific illustrations are provided to support the given results.

[1] Kiran Kumar Behera and A. Swaminathan, Biorthogonal rational functions of R_{II} type, Proc. Amer. Math. Soc., (2019), https://doi.org/10.1090/proc/14443, 13 pages.

[2] M. E. H. Ismail and D. R. Masson, Generalized orthogonality and continued fractions, J. Approx. Theory **83** (1995), no. 1, 1–40.
[3] L. Velázquez, Spectral methods for orthogonal rational functions, J. Funct. Anal. **254** (2008), no. 4, 954–986.
[4] A. Zhedanov, Biorthogonal rational functions and the generalized eigenvalue problem, J. Approx. Theory **101** (1999), no. 2, 303–329.

On π_N-coherent pair with index M and order (m,k) of orthogonal polynomial sequences

08.15 **Dieudonne Mbouna**
(*University of Coimbra, Portugal*)
Time: Friday 26.07., 11:30 - 12:00, Room HS 3

Abstract: Let M and N be non-negative integer numbers, π_N a monic polynomial of degree N, and $(P_n)_{n\geq 0}$ and $(Q_n)_{n\geq 0}$ two monic orthogonal polynomial sequences such that their normalized derivatives of orders m and k (respectively) satisfy

$$\pi_N(x)P_n^{[m]}(x) = \sum_{j=n-M}^{n+N} r_{n,j}Q_j^{[k]}(x)$$

for all $n = 0, 1, 2, \ldots$, where each $r_{n,j}$ is a complex number independent of x. It is shown that under some natural constraints, both $\{P_n\}_{n\geq 0}$ and $\{Q_n\}_{n\geq 0}$ belong to the semiclassical orthogonal polynomials class. In addition we show that the corresponding linear functionals with respect to which $\{P_n\}_{n\geq 0}$ and $\{Q_n\}_{n\geq 0}$ are orthogonal, are also connected by a rational modification (in the distributional sense). This leads to the concept of π_N-coherent pair with index M and order (m,k), as another generalization of the notion of coherent pair of measures introduced by A. Iserles, P. E. Koch, S. P. Nørsett, and J. M. Sanz-Serna [J. Aprox. Theory **65** (1991) 151–175], and subsequently generalized by several authors.

This is a joint work with Renato Alvarez-Nodarse, Kenier Castillo and José Carlos Petronilho.

MS09: Extremal polynomials and almost periodicity

Organizers: Jacob S. Christiansen (*Lund University, Sweden*)
Benjamin Eichinger (*Rice University, Houston, USA*)
Tom VandenBoom (*Yale University, New Haven, USA*)

In this mini-symposium we aim to discuss recent advances in the field of extremal polynomials, such as orthogonal and Chebyshev polynomials. New developments for the associated operators (given by, e.g., Jacobi or CMV matrices) as well as for continuous Schrödinger operators and canonical systems are naturally included. We are particularly interested in situations where almost periodicity occurs.

Bernstein-type inequalities and geometric function theory

09.01 **Sergei Kalmykov**
(*Shanghai Jiao Tong University, Shanghai, China*)
Time: Wednesday 24.07., 10:30 - 11:00, Room HS 4

Abstract: We will mainly discuss results concerning Bernstein- and Markov-type inequalities for polynomials and rational functions with restriction on zeros or curved majorants on the interval $[-1,1]$ or an arc. Methods of proofs are based on applying geometric function theory of complex variables. Special attention will be paid to cases of equality, construction of extremal polynomials and rational functions and mapping properties of these functions.

Lower bounds for extremal polynomials

09.02 **Maxim Zinchenko**
(*University of New Mexico, Albuquerque, USA*)
Time: Wednesday 24.07., 11:00 - 11:30, Room HS 4

Abstract: In this talk I will discuss lower bounds on the norms of L_p extremal polynomials. This work is an attempt to find sharp lower bounds that parallel the lower bounds for Chebyshev polynomials.

Periodic ellipsoidal billiards and extremal polynomials

09.03 **Vladimir Dragovic**
(*University of Texas at Dallas, USA*)
Time: Wednesday 24.07., 11:30 - 12:00, Room HS 4

Abstract: A comprehensive study of periodic trajectories of the billiards within ellipsoids in the d-dimensional Euclidean space is presented. The novelty of the approach is based on a relationship established between the periodic billiard trajectories and the extremal polynomials of the Chebyshev type on the systems of d intervals on the real line. The case study of trajectories of small periods $T, d \le T \le 2d$ is given. In particular, it is proven that all d-periodic trajectories are contained in a coordinate-hyperplane and that for a given ellipsoid, there is a unique set of caustics which generates $d+1$-periodic trajectories. A complete catalog of billiard trajectories with small periods is provided for $d = 2$ and 3. This is a joint work with Milena Radnovic.

Chebyshev polynomials on circular arcs

09.04 **Klaus Schiefermayr**
(*University of Applied Sciences Upper Austria, Wels, Austria*)
Time: Wednesday 24.07., 12:00 - 12:30, Room HS 4

Abstract: In this talk, we give an explicit representation of the Chebyshev polynomial on a given circular arc

$$A_\alpha := \{z \in \mathbb{C} : |z| = 1, -\alpha \le \arg(z) \le \alpha\}, \qquad 0 < \alpha \le \pi,$$

(a problem which was first considered in [1]), which is done in two steps: In the first step, following [2], we give an explicit representation of the Chebyshev polynomial (of degree N) on A_α in terms of the Chebyshev polynomial with respect to the weight function $w(x) := 1$ (for N even) and $w(x) := \sqrt{1-x^2}$ (for N odd) on the two real intervals $[-1,-a] \cup [a,1]$, where $a := \cos(\frac{\alpha}{2})$. For this representation, we will need the mapping $z \mapsto \frac{1}{2}\Big(\sqrt{z} + \frac{1}{\sqrt{z}}\Big)$ which maps $\{z \in \mathbb{C} : |z| = 1,\ \mathrm{Im}\{z\} \geq 0\}$ bijectively onto the interval $[0,1]$. In the second step, these Chebyshev polynomials (with respect to $w(x) := 1$ and $w(x) := \sqrt{1-x^2}$) are represented with the help of Jacobian elliptic and theta functions. These representations go back to [3] and [4]. The talk is based on the paper [5].

[1] J.-P. Thiran and C. Detaille, *Chebyshev polynomials on circular arcs in the complex plane*, Progress in approximation theory, Academic Press, 1991, pp. 771–786.

[2] F. Peherstorfer and K. Schiefermayr, *On the connection between minimal polynomials on arcs and on intervals*, in “Functions, Series, Operators” (Budapest, 1999), János Bolyai Math. Soc., Budapest, 2002, pp. 339–356.

[3] N.I. Akhiezer, *Über einige Funktionen, die in gegebenen Intervallen am wenigsten von Null abweichen*, Bull. Soc. Phys.-Math. Kazan, III. Ser. **3** (1928), 1–69 (in German).

[4] E.I. Krupickiĭ, *On a class of polynomials with least deviation from zero on two intervals*, Dokl. Akad. Nauk SSSR **138** (1961), 533–536.

[5] K. Schiefermayr, *Chebyshev polynomials on circular arcs*, to appear in Acta Sci. Math. (Szeged).

Continued fraction expansions and generalized indefinite strings

09.05 **Jonathan Eckhardt**
(*Loughborough University, UK*)
Time: Thursday 25.07., 10:30 - 11:00, Room HS 4

Abstract: Stieltjes continued fraction expansions play a decisive role in the solution of the inverse spectral problem for Krein strings. Certain continued fractions of a modified form correspond in the same way to generalized indefinite strings. I will discuss under which conditions Herglotz-Nevanlinna functions allow such an expansion and use this to solve the inverse spectral problem for generalized indefinite strings with coefficients supported on a discrete set. These results are related to the Hamburger moment problem as well as multi-soliton solutions of particular integrable wave equations.

Martin functions of Fuchsian groups and character automorphic subspaces of the Hardy space in the upper half plane

09.06 **Peter Yuditskii**
(*Johannes Kepler University, Linz, Austria*)
Time: Thursday 25.07., 11:00 - 11:30, Room HS 4

Abstract: We establish exact conditions for non triviality of all subspaces of the standard Hardy space in the upper half plane, that consist of the character automorphic functions with respect to the action of a discrete subgroup of $SL_2(\mathbb{R})$. Such spaces are the natural objects in the context of the spectral theory of almost periodic differential operators and in the asymptotics of the approximations by entire functions. A naive idea: it should be completely parallel to the celebrated Widom characterization for Hardy spaces on Riemann surfaces with a minor modification, namely, one has to substitute the Green function of the domain with the Martin function. Basically, this is correct, but…

The talk is based on a joint work with Aleksandr Kheifets. Supported by the Austrian Science Fund FWF, project no: P29363-N32.

Szegő condition and scattering for Dirac operators

09.07 **Roman Bessonov**
(*Saint Petersburg State University and PDMI, Russia*)
Time: Thursday 25.07., 11:30 - 12:00, Room HS 4

Abstract: A classical fact of scattering theory for one-dimensional Dirac operator is the existence of

strong wave operators in the case where the potential of the operator is absolutely integrable over the real line. Much more deep results by Christ, Kiselev (2002) and Denisov (2004) establish the existence of wave operators for potentials in L^p for $1 < p < 2$ and $p = 2$, correspondingly. In all cases the spectral measure of the Dirac operator under consideration belongs to the Szegő class on the real line: its density coincides with the absolute value of an outer function. We show that this condition always imply existence of wave operators, which allows us to describe a broad class of "large" potentials for which the wave operators exist. The work is supported by grant RScF 19-11-00058.

Schrödinger operators with substitutive potentials beyond linear complexity

09.08 **Philipp Gohlke**
(*Universität Bielefeld, Germany*)
Time: Thursday 25.07., 12:00 - 12:30, Room HS 4

Abstract: Tight-binding Schrödinger operators with potentials generated by primitive substitutions have been studied extensively in the past. Typically, these models exhibit singular continuous spectra of Lebesgue measure zero. We consider classes of substitutional systems that go beyond both minimality and linear complexity and explore some of the novel spectral phenomena that can occur. This is joint work with B. Eichinger.

Periodic coordinates and a magic formula for finite-gap CMV matrices

09.09 **Benjamin Eichinger**
(*Rice University Houston, Texas, USA*)
Time: Thursday 25.07., 15:30 - 16:00, Room HS 4

Abstract: We prove a bijective unitary correspondence between 1) the isospectral torus of almost-periodic, absolutely continuous CMV matrices having fixed finite-gap spectrum E and 2) periodic block-CMV matrices satisfying a *Magic Formula*. This latter class arises as E-dependent operator Möbius transforms of certain generating CMV matrices which are periodic up to a rotational phase; for this reason we call them "MCMV". Naturally, this has also consequences for the associated Schur functions. We show that for any Schur function associated to a finite-gap CMV matrix (and therefore with almost periodic Verblunsky coefficients) there exists a more general Nevanlinna-Pick interpolation problem with periodic interpolation data.

The talk is based on a joint work with J. S. Christiansen and T. VandenBoom.

Values distribution of almost periodic functions, spectral factorization and entire functions

09.10 **Wayne Lawton**
(*Siberian Federal University, Kransnoyarsk, Russia*)
Time: Thursday 25.07., 16:00 - 16:30, Room HS 4

Abstract: We recently derived an upper bound on the mean measure of the set where a nonzero Bohr almost periodic function f with bounded spectra has small modulus. This bound implies that $\log |f|$ is almost periodic in the sense of Besicovitch. We use this result to relate the Ahiezer factorization of an entire extension F of a nonnegative Bohr almost periodic function f with a bounded spectrum and the Helson-Lowdenslager spectral factorizations of its lift h to a compact group. We also discuss new classes of one dimensional quasicrystals related to both Bohr and Besicovitch almost periodic functions.

CMV block matrices for symmetric matrix measures on the unit circle

09.11 **Luis E. Garza**
(*Facultad de Ciencias, Universidad de Colima, Mexico*)
Time: Thursday 25.07., 16:30 - 17:00, Room HS 4

Abstract: Abstract: In this contribution, we study the relationship between the CMV block matrices associated with two symmetric positive definite matrix measures supported on the unit circle. We also consider certain transformations of an orthogonality matrix measure supported on the unit circle, and deduce connection formulas for the corresponding orthogonal families and their Verblunsky matrix coefficients.

Szegő minimum problem and Nevai's conjecture

09.12 **Anna Kononova**
(*Saint Petersburg State University, Russia*)
Time: Thursday 25.07., 17:00 - 17:30, Room HS 4

Abstract: We will present several quantitative results concerning with the Szegő minimum problem for classes of measure on the unit circle with divergent logarithmic integral. In particular, we refute a known Nevai's conjecture.

The talk is based on a joint work with A. Borichev and M. Sodin (arXiv:1902.00874, arXiv:1902.00872)

MS10: Potential theory and applications to orthogonal polynomials and minimal energy

Organizers: Peter Dragnev (*Purdue University Fort Wayne, Indiana, USA*)
Edward Saff (*Vanderbilt University, Nashville, Tennessee, USA*)

Recent applications of Potential Theory to the theory of orthogonal polynomials have allowed for significant advancement of the subject. Methods, such as the Riemann-Hilbert approach, for investigating asymptotic behavior of orthogonal polynomials include prominently the equilibrium measure of a compact set in the complex plane. Another important application of potential theory is to minimal energy problems on the sphere and other manifolds. Seemingly different, both of these areas of analysis explore the convergence properties of discrete potentials. It is the intention of the minisymposium is to provide a common bridge between them and allow for interchanging of ideas.

A new approach for solving equilibrium problems in potential theory

10.01 **Ana Matos**
(*Laboratoire Paul Painlevé, Université de Lille, France*)
Time: Thursday 25.07., 10:30 - 11:00, Room SH 02

Abstract: We are interested in computing the unknown density of an equilibrium problem in logarithmic potential theory where the support of the equilibrium measure is a finite union of distinct intervals. Inspired by a Riemann-Hilbert approach, we reduce the problem to the solution of a system of singular integral equations with Cauchy kernels.

After briefly recalling the well-studied polynomial approach, we will be interested in considering rational approximations of the solutions, expressed in a basis of orthogonal rational functions with prescribed poles. This new approach ensures stable computations. These approximations satisfy some interpolatory conditions. Inspired by the third Zolotareff problem, the poles and the interpolation points are chosen in such a way that we can ensure small errors. We will also discuss a link with recent performant algorithms like the multipole method.

Finally, our new error estimates will be confirmed by numerical results. This is joint work with B. Beckermann.

A family of entire functions connecting the Bessel function J_1 and the Lambert W function

10.02 **Christian Berg**
(*University of Copenhagen, Denmark*)
Time: Thursday 25.07., 11:00 - 11:30, Room SH 02

Abstract: At the 7th OPSFA, Copenhagen 2003, we posed the problem of determining the largest value $\alpha = \alpha^* > 0$ for which $f_\alpha(x) = e^\alpha - (1 + 1/x)^{\alpha x}$, $x > 0$ is a completely monotonic function, and it was noticed that $1 \le \alpha^* < 3$ and that graphs suggest that $\alpha^* > 2$. Numerical estimates given in [2] showed that $\alpha^* \approx 2.29965\,6443$.

We improve this result by combining Fourier analysis with complex analysis to find a family φ_α, $\alpha > 0$, of entire functions such that $f_\alpha(x) = \int_0^\infty e^{-sx}\varphi_\alpha(s)\,ds$ for $x > 0$.

We show that each function φ_α has an expansion in power series, whose coefficients are determined in terms of Bell polynomials. This expansion leads to several properties of the functions φ_α, which turn out to be related to the well known Bessel function J_1 when α is large, and to the Lambert W function when α is small.

On the other hand, by numerically evaluating the series expansion by using the alternating series test, we are able to show the behavior of φ_α as α increases from 0 to ∞ and to obtain a very precise approximation

of α^* such that $\varphi_\alpha(s) \geq 0$, $s > 0$, or equivalently, such that f_α is completely monotonic precisely for $0 < \alpha \leq \alpha^*$. We find $\alpha^* \approx 2.29965\,64432\,53461\,30332$.

The talk is based on the manuscript [1].

[1] C. Berg, E. Massa and A. P. Peron, *A family of entire functions connecting the Bessel function J_1 and the Lambert W function.* ArXiv:1903.07574.

[2] E. Shemyakova, S. I. Khashin and D. J. Jeffrey, *A conjecture concerning a completely monotonic function*, Computers and Mathematics with Applications **60** (2010), 1360–1363.

New bounds for the extreme zeros of classical orthogonal polynomials

10.03 **Geno Nikolov**
(*Sofia University "St. Kliment Ohridski", Bulgaria*)
Time: Thursday 25.07., 11:30 - 12:00, Room SH 02

Abstract: The zeros of classical orthogonal polynomials have been a topic of intensive investigation. There are many reasons for this interest, such as the nice electrostatic interpretation of the zeros of the Jacobi, Laguerre and Hermite polynomials, their important role as nodes of Gaussian quadrature formulae, as well as the key role these zeros play in the proofs of some classical inequalities.

Derivation of sharp upper and lower bounds for the extreme zeros is of particular interest. For this, powerful analytic and discrete techniques have been developed. Among them are Sturms comparison theorem for the zeros of solutions of second order differential equations, A. Markovs theorem on monotonicity of zeros of orthogonal polynomials in terms of the behavior of the weight function, the Hellmann-Feynman theorem on variation of eigenvalues of Hermitian matrices, the Obrechkoff theorem on Descartess rule of signs, the Wall-Wetzel theorem on eigenvalues of Jacobi matrices in terms of chain sequences, etc.

We apply the Euler-Rayleigh method to obtain some new bounds for the extreme zeros of the Jacobi (in particular, of Gegenbauer) and Laguerre polynomials. Typically, the comparison of the different estimates does not single out a "best" bound as these estimates depend on two or three parameters. We show that our bounds improve some of the best known bounds obtained recently.

The logarithmic and Riesz minimal energy problem on sets of revolution — new progress

10.04 **Johann Brauchart**
(*Technische Universität Graz, Austria*)
Time: Thursday 25.07., 12:00 - 12:30, Room SH 02

Abstract: Finding the logarithmic and Riesz equilibrium measure on a set of revolution which is not the sphere proves to be challenging even in the simplest cases of a finite cylinder (revolving line segments) or a circular torus (revolving circle). We discuss theoretical and numerical results. This is joint work with with Doug Hardin and Edward B. Saff.

Minimal energy problem on the sphere, equilibrium support, and quadrature domains

10.05 **Peter Dragnev**
(*Purdue University Fort Wayne, USA*)
Time: Thursday 25.07., 15:30 - 16:00, Room HS 5

Abstract: We consider the minimal logarithmic energy problem on the unit sphere in the presence of external field exerted by finitely many point masses. For relatively weak mass charges the equilibrium support is obtained by removing perfect spherical caps centered at the points. We shall characterize the

equilibrium support beyond such interactions and obtain a remarkable connection with quadrature domains in the complex plane. This is joint work with Alan Legg.

Spherical ensemble with two charges

10.06 **Juan G. Criado del Rey**
(*Katholieke Universitait Leuven, Belgium*)
Time: Thursday 25.07., 16:00 - 16:30, Room HS 5

Abstract: We study the weighted equilibrium measure associated to a logarithmic external field generated by two-point charges on the two-dimensional sphere. When the charges are small, the droplet is known to be the complement of two spherical caps. We describe the droplet when the charges are arbitrarily large, focusing on the symmetric case.

Potential theory on orthogonal polynomials arising from subnormal and hyponormal operators

10.07 **Nikos Stylianopoulos**
(*University of Cyprus, Cyprus*)
Time: Thursday 25.07., 16:30 - 17:00, Room HS 5

Abstract: The purpose of the talk is to show how tools from potential theory can be used in obtaining results for the distribution of zeros and the n-th root behavior, of certain orthogonal polynomials associated with subnormal and hyponormal operators.

Hyperuniformity on flat tori

10.08 **Tetiana Stepaniuk**
(*RICAM, Austrian Academy of Sciences, Linz, Austria*)
Time: Thursday 25.07., 17:00 - 17:30, Room HS 5

Abstract: In the talk we study hyperuniformity on flat tori. Hyperuniform point sets on the unit sphere have been studied by J. Brauchart, P. Grabner, W. Kusner and J. Ziee. It is shown that point sets which are hyperuniform for large balls, small balls or balls of threshold order on the flat tori are uniformly distributed. We also show that QMC-designs sequences for Sobolev classes and probabilistic point sets (with respect to jittered samplings) are hyperuniform.

MS11: Developments in q-series and the theory of partitions

Organizer: Ali Uncu (*Johannes Kepler University Linz, Austria*)

This mini-symposium is dedicated to discuss recent developments in the study of q-series and its implications on the theory of partitions in a broad perspective. We aim to welcome the representation of all the techniques used in the field such as series manipulations, basic hypergeometric transformations, modular forms, bijective combinatorics, etc.

New representations for $\sigma(q)$ via reciprocity theorems

11.01 **Koustav Banerjee**
(*Research Institute for Symbolic Computation, Johannes Kepler University, Linz, Austria*)
Time: Monday 22.07., 10:30 - 11:00, Room HS 4

Abstract: In this talk, we will see two new representations for Ramanujan's function $\sigma(q)$. The proof of the first one uses the three-variable reciprocity theorem due to Soon-Yi Kang and a transformation due to R.P. Agarwal while that of the second uses the four-variable reciprocity theorem due to George Andrews and a generalization of a recent transformation of Andrews, Schultz, Yee and the second author. The advantage of these representations is that they involve free complex parameters - one in the first representation, and two in the second. The partition theoretic interpretation has been already made in the context of three variable representation by Atul Dixit and Bibekananda Maji.

This is a joint work with Prof. Atul Dixit.

Modular properties of false theta functions

11.02 **Caner Nazaroglu**
(*Mathematical Institute, University of Cologne, Germany*)
Time: Monday 22.07., 11:00 - 11:30, Room HS 4

Abstract: False theta functions are functions that closely resemble classical theta functions, which despite this similarity do not have the modular properties that theta functions possess. They appear, for example, in the context of link invariants, W-algebras and also are closely related to mock modular forms. In this talk, I will describe modular properties of false theta functions and give a modular completion analogous to modular completions of mock modular forms. Finally, I will give an application of this machinery to derive a Rademacher type exact expression for the number of unimodal sequences and extend earlier work on their asymptotic properties.

New results on asymptotics and inequalities for partition functions

11.03 **Alexandru Ciolan**
(*Mathematical Institute, University of Cologne, Germany*)
Time: Monday 22.07., 11:30 - 12:00, Room HS 4

Abstract: We present some recent results on asymptotics and inequalities for (over)partitions. In the first part of this talk we prove a conjecture by Bringmann and Mahlburg (2012), which says that a large enough number n has more partitions into squares with an even number of parts than with an odd number of parts if n is even, and conversely if n is odd. In the second part we compute asymptotics for $\overline{N}(a, c, n)$, the number of overpartitions of n with a number of parts congruent to a modulo c and use these asymptotics to prove some inequalities on overpartition ranks conjectured by Ji, Zhang and Zhao (2018), and Wei and Zhang (2018).

Symbolic Summation, difference ring algorithms and q-applications

11.04 **Carsten Schneider**
(*Research Institute for Symbolic Computation, Johannes Kepler University, Linz, Austria*)
Time: Monday 22.07., 12:00 - 12:30, Room HS 4

Abstract: In the last years the difference ring approach for symbolic summation has been pushed forward substantially for multiple sums defined over hypergeometric products. Due to its generality, this approach also works for the q-hypergeometric, multibasic or mixed multibasic case. But so far it has been neglected up to some toy examples. In this talk I will report on new developments in cooperation with Jakob Ablinger and Ali K. Uncu to push forward the difference ring machinery for the q-world. Examples will be presented in which, e.g., q-trinomial coefficients arise.

Enumerating simultaneous core partitions into k distinct parts

11.05 **Hannah Burson**
(*University of Illinois at Urbana-Champaign, USA*)
Time: Monday 22.07., 15:30 - 16:00, Room HS 4

Abstract: In 2016, using a combinatorial bijection with certain abaci diagrams, Nath and Sellers enumerated $(s, ms \pm 1)$-core partitions into distinct parts. In this talk, we explain new generalizations of this theorem, with a focus on a generating polynomial that enumerates simultaneous core partitions by the number of parts.

This work is joint with Simone Sisneros-Thiry and Armin Straub.

On a continued fraction of Ramanujan

11.06 **Gaurav Bhatnagar**
(*University of Vienna, Austria*)
Time: Monday 22.07., 16:00 - 16:30, Room HS 4

Abstract: We study a continued fraction due to Ramanujan, that he recorded as Entry 12 in Chapter 16 of his second notebook. It is presented in Part III of Berndt's volumes on Ramanujan's notebooks. We give two alternate approaches to proving Ramanujan's Entry 12, one using a method of Euler, and another using the theory of orthogonal polynomials. We consider a natural generalization of Entry 12 suggested by the theory of orthogonal polynomials.

This is joint work with Mourad Ismail.

Some q-series conjectures related to Rogers-Ramanujan type identities of Kanade and Russell

11.07 **Chris Jennings-Shaffer**
(*Mathematical Institute, University of Cologne, Germany*)
Time: Monday 22.07., 16:30 - 17:00, Room HS 4

Abstract: In 2014, and in follow-up work in 2018, Kanade and Russell conjectured a large number of identities of the Rogers-Ramanujan type. Recently, in joint work with Kathrin Bringmann and Karl Mahlburg, we proved seven of their conjectures. Rather than focus on the proven identities, here we discuss some conjectural identities for certain q-hypergeometric series that came out of a reduction of other conjectures of Kanade and Russell. In these cases, the series can be expressed as ${}_2\phi_1$ series, rather than multi-sums, and the result should be a simple infinite product. While this sounds quite simple, somehow a full proof has eluded us.

Modular functions and computer algebra

11.08 **Peter Paule**
(*Research Institute for Symbolic Computation, Johannes Kepler University, Linz, Austria*)
Time: Monday 22.07., 17:00 - 17:30, Room HS 4

Abstract: The talk reports on recent progress concerning computer algebra related to modular functions. Examples are a proof of the Weierstrass gap theorem without using the Riemann-Roch formula, or a new algorithmic framework for proving Ramanujan's celebrated congruences for partition numbers.

Most of the results presented arose in joint work with Silviu Radu (RISC).

Exponential function on nonuniform lattices and solutions to some q-indeterminate moment problems

11.09 **Maurice Kenfack Nangho**
(*Department of Mathematics and Computer Science, University of Dschang, Cameroon*)
Time: Tuesday 23.07., 10:30 - 11:00, Room HS 4

Abstract: We develop analogs of exponential and trigonometric functions (including the basic exponential function) and derive their fundamental properties: addition formula, positivity, reciprocal and fundamental relations of trigonometry. We establish a binomial theorem provide a formula for computing the nth-derivatives for analytic functions on nonuniform lattices (q-quadratic and quadratic variables). We also develop solutions to q-indeterminate moment problems related with Askey-Wilson polynomials. This talk is based on [1,2].

[1] M. Kenfack Nangho, M. Foupouagnigni and W. Koepf: On exponential and trigonometric functions on nonuniform lattices, Ramanujan Journal, Volume 49, Issue 1, May 2019, Pages 1–37.

[2] M. Kenfack Nangho and K. Jordaan: A characterization of Askey-Wilson polynomials, Proceedings of the AMS, Volume 147, Number 6, June 2019, Pages 2465–2480.

Using geometry for computing q-series

11.10 **Zafeirakis Zafeirakopoulos**
(*Gebze Technology University, Turkey*)
Time: Tuesday 23.07., 11:00 - 11:30, Room HS 4

Abstract: Computing generating functions of objects defined by linear constraints (e.g., the majority of integer partition families) is an old but interesting game. Many counting generating functions have nice representations as q-series and all kinds of amazing identities or relations emerge. On the other hand, the full (multivariate) generating function contains more information, but it rarely gives rise to nice relations. Using polyhedral geometry, we can express the rational generating function for a set of combinatorial objects (e.g., solid partitions on a cube) as a sum of symbolic cones. Symbolic cones are a "multivariate" representation that does not suffer from the usual combinatorial explosion during computation, thus can be used for computing q-series.

In this talk, we will present the notion of symbolic cone, why it is computationally and conceptually preferable over rational functions and how to obtain q-series using Polyhedral Omega.

Multisums related to Rogers–Ramanujan type identities and their refinements

11.11 **Ali Kemal Uncu**
(*Research Institute for Symbolic Computation, Johannes Kepler University, Linz, Austria*)
Time: Tuesday 23.07., 11:30 - 12:00, Room HS 4

Abstract: There has been a recent influx of Andrews–Gordon identities like multi nested q-series representations of the generating functions for the number of partitions with certain difference conditions. We

will present how one can refine these generating functions by imposing bounds on the largest part of the related partitions. These refinements will lead to many polynomial identities.

The Deng-Yang conjecture

11.12 **Mourad E. H. Ismail**
(*University of Central Florida, USA*)
Time: Tuesday 23.07., 12:00 - 12:30, Room HS 4

Abstract: Ding and Yang conjectured that the q-normal density (the weight function for q-Hermite polynomials suitably normalized) tends to a sum of two delta functions. We prove their conjecture and show that this a general property of most of the q-orthogonal polynomials. This is based on joint work with D. Dai and X. W. Wang.

Ruscheweyh-type starlike functions of complex order associated with q-difference operator

11.13 **Asena Çetinkaya**
(*Istanbul, Turkey*)
Time: Tuesday 23.07., 15:30 - 16:00, Room HS 4

Abstract: Quantum calculus or q-calculus dates back to Leonhard Euler (1707–1783) and Carl Gustav Jacobi (1804–1851). But q-calculus became popular only after its usefulness in quantum mechanics after 1905 paper by Albert Einstein. In 1909 and 1910 Jackson initiated in-depth study of q-calculus. He was the first to develop the q-integral and q-derivative in a systematic way. The great interest is due to its applications in various branches of mathematics and physics, as for example, in the areas of ordinary fractional calculus, orthogonal polynomials, basic hypergeometric functions, combinatorics.

In this paper, we investigate three new subclasses of Ruscheweyh-type starlike functions of complex order associated with q- difference operator. We investigate inclusion theorem, sufficient coefficient estimates, distortion bounds and radius of starlikeness of these subclasses. Further, we obtain partial sums of our classes.

Some new Ramanujan–Kohlberg identities

11.14 **Nicolas Smoot**
(*Research Institute for Symbolic Computation, Johannes Kepler University, Linz, Austria*)
Time: Tuesday 23.07., 16:00 - 16:30, Room HS 4

Abstract: Ramanujan's identities involving the generating functions for $p(5n + 4)$ and $p(7n + 5)$ are considered to be among his most beautiful results. These were shown by Kolberg to be special cases of a larger class of relationships expressing generating functions for $p(mn + j)$ in terms of eta quotients. The form of these identities is prevalent throughout the theory of partitions. They are useful in the verification of families of partition congruences, as well as in the study of certain conjectures in the theory of modular functions. In 2014 Silviu Radu developed an algorithm to compute the Ramanujan-Kolberg identities inherent in various arithmetic functions. We have fully implementated this algorithm with Mathematica. We will show some interesting examples found using our implementation, with a focus on partition congruences. We include some new results, as well as some interesting improvements on previous results.

A q-analogue for Euler's $\zeta(6) = \pi^6/945$

11.15 **Ankush Goswami**
(*University of Florida, USA / RISC, Johannes Kepler University, Linz, Austria*)
Time: Tuesday 23.07., 16:30 - 17:00, Room HS 4

Abstract: Recently, Z.-W. Sun obtained q-analogues of Euler's formula for $\zeta(2)$ and $\zeta(4)$. Sun's formula

were based on identities satisfied by triangular numbers and properties of Euler's q-Gamma function. In this talk, we discuss a q-analogue of $\zeta(6) = \pi^6/945$. Indeed, we have been able to obtain q-analogues of Euler's formula for $\zeta(2k)$, $k = 4, 5, \ldots$ (the general case). However, it is to be noted here that the case $k = 3$ or the q-analogue of $\zeta(6)$ is striking as it leads to very interesting connections. Also, the q-analogue of $\zeta(6)$ is the first non-trivial case where we see the occurence of a certain "extra" term which goes to zero as $q \to 1$ from inside the unit disk. We will also shed some light on this extra term.

Proofs of some q-product identities conjectured by Merca

11.16

Silviu Radu
(*Research Institute for Symbolic Computation, Johannes Kepler University, Linz, Austria*)
Time: Tuesday 23.07., 17:00 - 17:30, Room HS 4

Abstract: In the present talk we show how to prove several infinite q-product identities using modular functions. These q-product identities have been conjectured by Mircea Merca. This is joint work with Christian Krattenthaler and Mircea Merca.

MS12: General session for contributed talks

Integrals of Mellin–Barnes type

12.01 **Gopal Srinivasan**
(*Department of Mathematics, Indian Institute of Technology Bombay, India*)
Time: Monday 22.07., 10:30 - 11:00, Room HS 6

Abstract: Integrals involving ratios of products of gamma functions along vertical lines in the right half plane have been important since their appearance in 1890s featuring prominently in analytic number theory (in the works of Cahen, Pincherle and others) and also formed a point of departure for Barnes for his development of the theory of hypergeometric functions. The Meier G-functions are also representable as integrals of Mellin-Barnes type. These integrals feature in the work of Hecke in connection with certain problems in algebraic number theory. In this talk we shall look at these integrals from the point of view of pull backs of distributions in the sense of Laurant Schwartz via submersive maps deriving many classical well-known identities as corollaries.

Discrete variations on an old special functions theme

12.02 **Ciprian-Sorin Acatrinei**
(*NIPNE, Bucharest, Romania*)
Time: Monday 22.07., 11:00 - 11:30, Room HS 6

Abstract: A discretization scheme is discussed for field theories defined over $2+1$ and $3+1$ dimensional spaces, with particular emphasis on the special-functions-type solutions of their equations of motion. The space-time coordinates are assumed to be operators forming a non-commutative algebra; in the representation chosen, the radial coordinate becomes discrete, whereas the dependence on the other coordinates transfers to nonlocal field correlations. The discrete equations of motion and their complete solutions are presented in detail, together with their continuum/commutative limit. In $2+1$ dimensions one obtains a discrete extension of the Bessel and Neumann functions, passing through less known aspects of the Laguerre polynomials recurrence relation. In $3+1$ dimensions the relevant recurrence relation is the one satisfied by the Hahn polynomials.

Symbolic evaluation of *hp*-FEM element matrices on simplices

12.03 **Tim Haubold**
(*University of Hannover, Germany*)
Time: Monday 22.07., 11:30 - 12:00, Room HS 6

Abstract: In this talk we consider high-order finite element discretizations of linear elliptic boundary value problems. Following e.g. [1,2] a set of hierarchic basis functions on simplices is chosen. For an affine simplicial triangulation this leads to a sparse stiffness matrix. Moreover the L_2-inner product of the interior basis functions is sparse with respect to the polynomial order p. The construction relies on a tensor-product based construction with properly weighted Jacobi polynomials.

In this talk we present algorithms which compute the remaining non zero entries of mass- and stiffness matrix in optimal arithmetical complexity. In order to obtain this result, recursion fomulas based on symbolic methods [3] are used. The presented techniques can be applied not only to scalar elliptic problems in H^1 but also for vector valued problems in $H(\mathrm{div})$ and $H(\mathrm{curl})$, where an explicit splitting of the higher-order basis functions into solenoidal and non-solenoidal ones is used.

[1] Beuchler, Pillwein, Schöberl, Zaglmayr: *Sparsity Optimized High Order Finite Element Functions on Simplices*, 2012.
[2] Karniadakis, Sherwin: *Spectral/HP Element Methods for CFD*, 1999.
[3] Kauers: *SumCracker – A Package for Manipulating Symbolic Sums and Related Objects*, 2006.

Orthogonal polynomials arising from the expansion of first degree polynomials

12.04 **Shara Lalo**
(*Derby, UK*)
Time: Monday 22.07., 12:00 - 12:30, Room HS 6

Abstract: Expansions of univariate polynomials of the first degree with real and complex coefficients; naturally produce many well-known polynomials such as; Chebyshev, Fibonacci, Pell, Jacobsthal, Fermat polynomials and many new orthogonal polynomials. We will present new differential equations related to orthogonal polynomials arising from the expansion of first-degree polynomials. The roots, weights, generating functions and equivalent Rodriguez type formulas for the orthogonal polynomials will also be presented. This is joint work with Zagros Lalo.

Hook length property of d-complete posets via q-integrals

12.05 **Meesue Yoo**
(*Dankook University, South Korea*)
Time: Tuesday 23.07., 17:00 - 17:30, Room HS 5

Abstract: In this work, we prove the hook length property of the d-complete posets using the q-integral technique developed by Kim and Stanton. For a non-negative integer n, the generating function for the number of partitions of n with no more than k parts, $p_k(n)$, is given by

$$\sum_{n=0}^{\infty} p_k(n) q^n = \prod_{i=1}^{k} \frac{1}{1-q^i}.$$

Considering a partition of n with no more than k parts as an order-reversing map from a k-element chain to the set of non-negative integers such that the sum of images equals to n, Stanley extended this concept of partition and defined *P-partitions* of n. Then Stanley proved that the P-partition generating function for shapes has the hook length property. Proctor and Peterson figured out that the d-complete posets satisfy the hook length property, and Proctor showed that any connected d-complete poset P can be uniquely decomposed into a slant sum of one element posets and irreducible components. Furthermore, he classified 15 disjoint classes of irreducible components and showed that these 15 disjoint classes exhaust the set of all irreducible components. We show that the P-partition generating function for each irreducible d-complete poset can be written as a q-integral and prove the hook length property of them by computing the q-integrals explicitly. This is a joint work with Jang Soo Kim.

Recurrence equations involving different orthogonal polynomial sequences

12.06 **Aletta Jooste**
(*University of Pretoria, South Africa*)
Time: Wednesday 24.07., 10:30 - 11:00, Room SH 02

Abstract: Every sequence of real polynomials $\{p_n\}_{n=0}^{\infty}$, orthogonal with respect to a positive weight function $w(x)$ on the interval (a, b), satisfies a three-term recurrence equation. We discuss the role played by the polynomials *associated* to p_n, especially as coefficient polynomials in the three-term recurrence equation involving polynomials p_n, p_{n-1} and $p_{n-m}, m \in \{2, 3, \ldots, n-1\}$. Furthermore, we show how Christoffel's formula is used to obtain mixed three-term recurrence equations involving the polynomials p_n, p_{n-1} and $g_{n-m,k}, m \in \{2, 3, \ldots, n-1\}$, where the sequence $\{g_{n,k}\}_{n=0}^{\infty}, k \in \mathbb{N}_0$, is orthogonal with respect to $c_k(x)w(x) > 0$ on (a, b) and c_k is a polynomial of degree k in x. The equations obtained can be used to study the location of the zeros of the appropriate polynomials.

Completely monotonic Fredholm determinants

12.07 **Ruiming Zhang**
(*College of Science, Northwest A&F University, Yangling, Shaanxi, China*)
Time: Wednesday 24.07., 11:00 - 11:30, Room SH 02

Abstract: This talk is based on a joint work with Professor Mourad Ismail. In this talk we discuss some monotonicity questions related to Fredholm matrices and operators. A function $f(x)$ is called completely monotonic if $(-1)^m f^{(m)}(x) > 0$. It is known that the expectation of having m eigenvalues of a random Hermitian matrix in an interval is a multiple of $(-1)^m$ times the m-th derivative of a Fredholm determinant at $\lambda = 1$. In this work we extend the positivity to half-real line $(-\infty, 1]$, and we also study the completely monotonicity of some special functions which arise as Fredholm determinants.

A generalization of the Silvester-Kac matrix

12.08 **Mikhail Tyaglov**
(*Shanghai Jiao Tong University, China*)
Time: Wednesday 24.07., 11:30 - 12:00, Room SH 02

Abstract: Recently [2], C. da Fonseca et al considered a model for deposition and evaporation on discrete cells of a finite array of any dimension that led to a matrix equation involving a Sylvester-Kac type matrix. They found the eigenvalues and eigenvectors of that matrix and generalized some results of R. Askey [1] and O. Holtz [3]. In this talk, we discuss a somewhat novel approach that allows to generalize the results of all previous authors. More exactly, we find the eigenvalues and eigenvectors of the following $N \times N$ matrix:

$$\begin{pmatrix} 0 & \alpha & 0 & \dots & 0 & 0 & 0 \\ -N\gamma & \beta & 2\alpha & \dots & 0 & 0 & 0 \\ 0 & -(N-1)\gamma & 2\beta & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & (N-2)\beta & (N-1)\alpha & 0 \\ 0 & 0 & 0 & \dots & -2\gamma & (N-1)\beta & N\alpha \\ 0 & 0 & 0 & \dots & 0 & -\gamma & N\beta \end{pmatrix},$$

where α, β, and γ are arbitrary complex numbers, and $\gamma \neq 0$.

[1] R. Askey, Evaluation of Sylvester type determinants using orthogonal polynomials, in: H.G.W. Begehr, et al. (Eds.), Advances in Analysis, World Scientific, Hackensack, NJ, 2005, pp. 1–16.
[2] C. da Fonseca, D. Mazilu, I. Mazilu, T. Williams, The eigenpairs of a Sylvester-Kac type matrix associated with a simple model for one-dimensional deposition and evaporation, Appl. Math. Lett., 26, no. 12, pp. 1206–1211.
[3] O. Holtz, Evaluation of Sylvester type determinants using block-triangularization, in: H.G.W. Begehr, et al. (Eds.), Advances in Analysis, World Scientific, Hackensack, NJ, 2005, pp. 395–405.

The fractional Green's function by Babenko's approach

12.09 **Chenkuan Li**
(*Brandon University, Brandon, Manitoba, Canada*)
Time: Wednesday 24.07., 12:00 - 12:30, Room SH 02

Abstract: The goal of the current work is to derive the fractional Green's function for the first time in the distributional space $\mathcal{D}'(R^+)$ for the following fractional-order differential equation with constant coefficients

$$a_n u^{(\beta_n)}(x) + a_{n-1} u^{(\beta_{n-1})}(x) + \cdots + a_1 u^{(\beta_1)}(x) + a_0 u^{(\beta_0)}(x) = g(x).$$

Our new technique is based on Babenko's approach, without using any integral transforms, such as the Laplace transform, and the Mittag-Leffler functions. The results obtained are not only simpler but also

more generalized than classical ones as they deal with distributions in Schwartz's sense. Furthermore, we provide several interesting applications of solving the fractional differential and integral equations by showing the convergence of double series based on gamma functions.

New degenerate Eulerian polynomials

12.10 **Orli Herscovici**
(*Department of Mathematics, Technion – Israel Institute of Technology, Haifa, Israel*)
Time: Friday 26.07., 10:30 - 11:00, Room HS 4

Abstract: We introduce a new generalization of the Eulerian polynomials based on the degenerate exponential function defined by Tsallis. Classical Eulerian polynomials can be defined by a few different generating functions. We present the generalizations of those generating functions, study properties of the degenerate Eulerian polynomials and present generalizations of some familiar identities which we have established in our preliminary work. Moreover, we give an explicit form of the coefficients of the degenerate Eulerian polynomials.

The Gamma function and its inverse

12.11 **David Jeffrey**
(*University of Western Ontario, London, Ontario, Canada*)
Time: Friday 26.07., 11:00 - 11:30, Room HS 4

Abstract: Some new results on expansions of the Gamma function will be presented. The functional inverse of the Gamma function has been little studied, although there are applications. The definition of the inverse will be discussed and its computation described.

Discretization of generalized Chebyshev polynomials of (anti)symmetric multivariate sine functions

12.12 **Adam Brus**
(*Czech Technical University, Prague, Czech Republic*)
Time: Friday 26.07., 11:30 - 12:00, Room HS 4

Abstract: The multivariate antisymmetric and symmetric trigonometric functions allow to generalize the classical Chebyshev polynomials to multivariate settings. The four classes of the multivariate polynomials, related to the symmetrized sine functions, are studied. For each of these polynomials, the weighted continuous and discrete orthogonality relations are shown. The related cubature formulas for numerical integration together with further model examples and properties of selected special cases are discussed.

Poster session

Semiclassical quasi-orthogonal polynomials. A general calculus approach

Roberto S. Costas Santos
(*Universidad de Alcalá, Madrid, Spain*)

Abstract: In this poster we present some new results related to semiclassical orthogonal polynomials. It is well-known that the classical orthogonal polynomials sequences (COPS) are characterized as these polynomials sequences whose derivative difference or q-difference is also orthogonal. The main aims of this poster is twofold:

(i) Given a COPS (p_n) we introduce a new polynomial sequence which we denote by nested kernel polynomials associated to (p_n) that will be up to a constant equal to the derivative difference or q-difference of the original sequence.

(ii) We state a characterization theorem for semiclassical orthogonal polynomial sequences where such nested kernel polynomials will play a crucial role.

This is joint work with Paco Marcellán.

On modified Meixner-Pollaczek polynomials

Abey Kelil
(*University of Pretoria, South Africa*)

Abstract: We investigate certain properties of monic polynomials orthogonal with respect to modified Meixner-Pollaczek weight function. By applying an exponential modification of the classical Meixner-Pollaczek measure, we explore certain properties such as moments, some new recurrence relations, differential-recurrence equations, explicit representations, generating functions, and integral representations of the modified polynomials. Certain practical applications of these polynomials are also investigated.

Negative and positive polynomial expansion theorems

Shara Lalo and Zagros Lalo
(*London & Derby, UK*)

Abstract: We present novel methods for negative and positive polynomial expansions of univariate polynomials. The equations we have derived can perform expansions of polynomials for any exponent in the complex number set (negative or positive). The equations are very general and can expand any polynomial even when the polynomial has some missing terms, and the coefficients in the input polynomial can have any value, real or complex. The method used in formulating the new mathematical equations is scalable and can be used to derive equations for expansions of any higher degree polynomial. The main advantage of the new equations is that any coefficient in the expansion can be obtained without the need to perform the entire expansion; this will be useful in problems involving probability and combinatorics.

Limit relations between d-orthogonal polynomials

Imed Lamiri
(*Higher School of Sciences and Technology, Sousse University, Tunisia*)

Abstract: Recently, it has been introduced the so-called d-Askey-scheme of d-orthogonal polynomials, as a similar table of the well-known Askey-scheme, which is a graphical scheme describing the families of hypergeometric orthogonal polynomials. An interesting aspect of this table is the limit transitions between some of these polynomials. In this work, we focused our interest on this aspect for some d-orthogonal polynomials. In fact, this work is composed of two parts, in the first one, we use a method based on a generating function to provide asymptotic expansions and limit relations linking between the d-Hermite polynomials and the d-Laguerre, d-Meixner, d-Charlier and d-Gegenbauer polynomials respectively, which represent an asymptotic study of the involved d-orthogonal polynomials. Moreover, we singled out the case $d = 2$, for which we include some numerical experiments illustrating the accuracy of the given approximations. All obtained results are reduced, in the case $d = 1$, to well known approximations involving orthogonal polynomials. In the second part of this work, we gave various limit relations linking between all levels of the 2-Askey scheme. To this end, we derive recurrence relations for all polynomials in the 2-Askey scheme, and we extend a method introduced by Koornwinder and based on a recurrence relation to derive the desired limit relations. This is joint work with Jihen Weslati.

The matching condition for higher dimensional Riemann-Hilbert problems

Leslie Molag
(*Katholieke Universitait Leuven, Belgium*)

Abstract: In a higher dimensional Riemann-Hilbert problem matching the local parametrix with the global parametrix is often a major issue. In the existing literature the approaches to obtain the matching for specific higher dimensional RHPs are quite technical in nature and these are only for 3×3 or 4×4 RHPs. For RHPs of an even higher dimension the complexity of these methods seems to increase drastically. I will present a result that should resolve the problem of the matching for higher dimensional RHPs in natural situations, making such technical approaches unnecessary in the future. I prove that, in a general setting, it is possible to obtain a double matching, that is, a matching condition on two circles instead of one circle.

Shift operators and Rodrigues formulas for matrix-valued discrete and q-orthogonal polynomials

Lucia Morey
(*National University of Córdoba, Argentina*)

Abstract: In this work, starting from a suitable matrix-valued analog of a Pearson equation, we find conditions for the existence of lowering and raising operators for matrix-valued orthogonal polynomials with respect to a discrete and a q-measure. In particular we use the shift operators to obtain, in a natural way, a Rodrigues formula, a structure formula and a second order difference or q-difference operator having the orthogonal polynomials as eigenfunctions. This approach allows us to study different types of matrix weights and to find families of matrix-valued orthogonal polynomials where the previous properties can be described explicitly.

Numerical evaluation of discrete Painlevé equations

Diego Ruiz-Antolín
(*University of Cantabria, Santander, Spain*)

Abstract: Many cases of discrete Painlevé equations arise from the study of the coefficients of the recurrence relations of certain orthogonal polynomials associated to semi-classical weights. A naive approach to the evaluation of the solutions of these equations could be the direct evaluation of the nonlinear recurrence relations that define them. However, there are multiple cases like the alternative discrete Painlevé I (alt-$\mathrm{dP_I}$) or the discrete Painlevé IV ($\mathrm{dP_{IV}}$) where the direct evaluation in both directions of the recurrence can be unstable for the nonnegative case. But there are also certain cases of discrete forms of Painlevé III that don't have this instability. We observe that the unstable solutions have as an initial point recesive solutions of certain linear differential equations like the case of alt-$\mathrm{dP_I}$ for which the initial point is a combination of Airy functions Ai(x). We are interested in both explaining the cause of these instabilities and

describing numerical and asymptotic methods to evaluate the nonnegative solutions of these recurrence relations when the direct evaluation is unstable. This is joint work with Alfredo Deaño Cabrera.

The dual orthogonal relations of q-discrete orthogonal polynomials derived by the (f, g)-inversion formula

Xinrong Ma
(*Department of Mathematics, Soochow University, Suzhou, China*)

Abstract: By means of three specific cases of the (f, g)-inversion formula [1, Theorem 1.3], we establish all dual forms of the orthogonality of orthogonal polynomials such as the little and big q-Jacobi, q-Racah, and q-Laguerre polynomials, including a generalization of the q-Laguerre polynomials. Further, we show that the Askey-Wilson q-beta integral represented in terms of the VWP-balanced ${}_8\phi_7$ series is just dual to the orthogonality of the Askey-Wilson polynomials.

[1] X. R. Ma, *An extension of Warnaar's matrix inversion*, Proc. Amer. Math. Soc. **133** (2005), 3179–3189.

Matrix-valued polynomials orthogonal with respect to exponential type weights

Bruno Eijsvoogel
(*Radboud University Nijmegen, Netherlands*)

Abstract: In recent joint work with Alfredo Deaño and Pablo Román we have studied a class of matrix weight functions supported on the real line and their corresponding orthogonal polynomials. Results contain non-linear recursions for the three term recurrence coefficients and a finite dimensional Lie algebra generated by differential ladder operators.

List of Participants

1	**Jakob Ablinger** RISC, Johannes Kepler University, Linz, Austria		
2	**Ciprian-Sorin Acatrinei** NIPNE, Bucharest, Romania	Mon 11:00, HS 6	p. 75
3	**Seamus Albion** The University of Queensland, Australia		
4	**Swaminathan Anbhu** Indian Institute of Technology, Roorkee, Uttarakhand, India	Fri 11:00, HS 3	p. 61
5	**Inês Aniceto** University of Southampton, UK	Mon 10:30, HS 5	p. 53
6	**Takashi Aoki** Kindai University, Japan	Mon 16:00, HS 5	p. 55
7	**Jorge Arvesú Carballo** Universidad Carlos III de Madrid, Spain	Mon 16:30, HS 3	p. 44
8	**Kam Cheong Au** The Hong Kong University of Science and Technology, Hong Kong		
9	**Cyril Banderier** CNRS / Université Paris Nord, France		
10	**Koustav Banerjee** RISC, Johannes Kepler University, Linz, Austria	Mon 10:30, HS 4	p. 70
11	**Ahmad Barhoumi** Indiana University IUPUI, USA	Wed 12:00, HS 3	p. 58
12	**Bernhard Beckermann** Lab. de Mathématiques Paul Painlevé, Université de Lille, France	Thu 12:00, HS 3	p. 59
13	**Nicolas Behr** IRIF, Université Paris Diderot, France	Mon 16:00, HS 6	p. 34
14	**Christian Berg** University of Copenhagen, Denmark	Thu 11:00, SH 02	p. 67
15	**Elías Berriochoa** Universidad de Vigo, Facultad de Ciencias Ourense, Spain		
16	**Roman Bessonov** Saint Petersburg State University and PDMI, Russia	Thu 11:30, HS 4	p. 64
17	**Gaurav Bhatnagar** Universität Wien, Vienna, Austria	Mon 16:00, HS 4	p. 71
18	**Asmus K. Bisbo** Gent University, Belgium		
19	**Petr Blaschke** Silesian University in Opava, Czech Republic	Mon 16:00, AM	p. 30
20	**Niels Bonneux** Katholieke Universitait Leuven, Belgium	Thu 16:00, AM	p. 26
21	**Thomas Bothner** King's College London, UK	Wed 09:00, AM	p. 17

22	**Amilcar Branquinho** Universidade de Coimbra, Portugal	Tue 17:00, HS 3	p. 47
23	**Johann Brauchart** Technische Universität Graz, Austria	Thu 12:00, SH 02	p. 68
24	**Adam Brus** Czech Technical University, Prague, Czech Republic	Fri 11:30, HS 4	p. 78
25	**Hannah Burson** University of Illinois at Urbana-Champaign, USA	Mon 15:30, HS 4	p. 71
26	**William Riley Casper** Louisiana State University, Baton Rouge, Louisiana, USA	Thu 15:30, AM	p. 26
27	**Andrew Celsus** University of Cambridge, UK	Wed 11:30, HS 3	p. 57
28	**Clemente Cesarano** Section of Mathematics – Uninettuno University, Roma, Italy	Tue 16:30, HS 3	p. 47
29	**Asena Çetinkaya** Istanbul, Turkey	Tue 15:30, HS 4	p. 73
30	**Hamza Chaggara** Sousse University, Tunisia	Mon 15:30, HS 3	p. 44
31	**Theodore Chihara** Purdue University Northwest, United States		
32	**Yong-Kum Cho** Chung-Ang University, Seoul, South Korea	Mon 12:00, AM	p. 30
33	**Jacob Christiansen** Lund University, Sweden		
34	**Seok-Young Chung** Chung-Ang University, South Korea		
35	**Óscar Ciaurri** Universidad de La Rioja, Spain		
36	**Alexandru Ciolan** Mathematical Institute, University of Cologne, Germany	Mon 11:30, HS 4	p. 70
37	**Peter A. Clarkson** University of Kent, Canterbury, UK	Fri 12:00, AM	p. 21
38	**Howard Cohl** National Institute of Standards and Technology, Gaithersburg, USA	Wed 10:30, AM	p. 23
39	**Roberto S. Costas Santos** Universidad de Alcalá, Madrid, Spain	Thu 17:00, AM Poster	p. 26 p. 79
40	**Juan G. Criado del Rey** Katholieke Universitait Leuven, Belgium	Thu 16:00, HS 5	p. 69
41	**Dan Dai** City University of Hong Kong	Mon 11:30, HS 5	p. 53
42	**Sourav Das** National Institute of Technology, Jamshedpur, Jharkhand, India	Mon 17:00, AM	p. 31
43	**Alfredo Deaño** University of Kent, UK	Wed 11:00, HS 3	p. 57
44	**Hadewijch De Clercq** Ghent University, Belgium	Thu 11:00, HS 6	p. 39

45	**Jan Derezinski** University of Warsaw, Faculty of Physics, Poland	Thu 11:30, AM	p. 25
46	**Manuel Domínguez de la Iglesia** Universidad Nacional Autónoma de México, Mexico	Mon 16:30, HS 6	p. 34
47	**Diego Dominici** Johannes Kepler University Linz, Austria		
48	**Peter Dragnev** Purdue University Fort Wayne, USA	Thu 15:30, HS 5	p. 68
49	**Vladimir Dragovic** University of Texas at Dallas, USA	Wed 11:30, HS 4	p. 63
50	**Kathy Driver** University of Cape Town, South Africa	Tue 10:30, AM	p. 32
51	**Michael Drmota** Technische Universität Wien, Austria	Tue 12:00, HS 5	p. 49
52	**Alexander Dyachenko** Keldysh Institute of Applied Mathematics, Moscow, Russia	Tue 11:30, HS 3	p. 46
53	**Anton Dzhamay** University of Northern Colorado, Greeley, USA	Wed 12:00, AM	p. 24
54	**Akihito Ebisu** Chiba Institute of Technology, Japan	Tue 15:30, AM	p. 22
55	**Jonathan Eckhardt** Loughborough University, UK	Thu 10:30, HS 4	p. 64
56	**Benjamin Eichinger** Rice University Houston, Texas, USA	Thu 15:30, HS 4	p. 65
57	**Bruno Eijsvoogel** Radboud University Nijmegen, Netherlands		
58	**Lidia Fernández** Universidad de Granada, Spain	Wed 10:30, HS 6	p. 37
59	**Leonardo Figueroa** Universidad de Concepción, Chile	Wed 11:00, HS 6	p. 38
60	**Galina Filipuk** University of Warsaw, Poland	Fri 10:30, HS 5	p. 27
61	**Ana Pilar Foulquié Moreno** Universidade de Aveiro, Portugal	Mon 16:00, HS 3	p. 44
62	**Avital Frumkin** Tel Aviv University, Israel		
63	**Luis E. Garza** Facultad de Ciencias, Universidad de Colima, Mexico	Thu 16:30, HS 4	p. 66
64	**Lino Gustavo Garza Gaona** Physics and Math Department, Universidad de Monterrey, Mexico	Tue 16:00, HS 6	p. 36
65	**Wolfgang Gawronski** University of Trier, Germany		
66	**Philipp Gohlke** Universität Bielefeld, Germany	Thu 12:00, HS 4	p. 65
67	**Ankush Goswami** RISC, Johannes Kepler University, Linz, Austria	Tue 16:30, HS 4	p. 73

68	**Wolter Groenevelt** Delft University of Technology, Netherlands	Thu 11:30, HS 6	p. 39
69	**Alan Groot** Katholieke Universitait Leuven, Belgium	Thu 11:00, HS 3	p. 58
70	**Yoshishige Haraoka** Department of Mathematics, Kumamoto University, Japan	Tue 17:00, AM	p. 23
71	**Tim Haubold** University of Hannover, Germany	Mon 11:30, HS 6	p. 75
72	**Ralf Hemmecke** RISC, Johannes Kepler University, Linz, Austria		
73	**Orli Herscovici** Technion – Israel Institute of Technology, Haifa, Israel	Fri 10:30, HS 4	p. 78
74	**Christopher J. Howls** University of Southampton, UK	Mon 11:00, HS 5	p. 53
75	**Weiying Hu** Department of Mathematics, City University of Hong Kong	Thu 11:00, AM	p. 25
76	**Hui Huang** University of Waterloo, Canada	Thu 11:00, HS 5	p. 51
77	**Erdal Imamoglu** Department of Mathematics, North Carolina State University, USA	Thu 12:00, HS 5	p. 52
78	**Sanjida Islam** University of Kent, United Kingdom		
79	**Mourad E. H. Ismail** University of Central Florida, USA	Tue 12:00, HS 4	p. 73
80	**Masahiko Ito** University of the Ryukyus, Nishihara, Okinawa Prefecture, Japan	Thu 12:00, HS 6	p. 40
81	**Katsunori Iwasaki** Hokkaido University, Japan	Tue 16:00, AM	p. 22
82	**Dragana Jankov Maširević** Josip Juraj Strossmayer University of Osijek, Croatia	Mon 11:30, AM	p. 29
83	**David Jeffrey** University of Western Ontario, London, Ontario, Canada	Fri 11:00, HS 4	p. 78
84	**Chris Jennings-Shaffer** Mathematical Institute, University of Cologne, Germany	Mon 16:30, HS 4	p. 71
85	**Antonio Jiménez Pastor** RISC, Johannes Kepler University, Linz, Austria	Mon 15:30, AM	p. 30
86	**Yuhui Jin** California Institute of Technology, Pasadena, USA		
87	**Lin Jiu** Dalhousie University, Halifax, Canada	Tue 11:30, HS 5	p. 49
88	**Fredrik Johansson** INRIA Bordeaux and Institut de Mathématiques de Bordeaux, France	Wed 12:00, HS 5	p. 50
89	**Aletta Jooste** University of Pretoria, South Africa	Wed 10:30, SH 02	p. 76
90	**Kerstin Jordaan** University of South Africa, Pretoria, South Africa	Thu 16:30, HS 3	p. 60

91	**Sergei Kalmykov** Shanghai Jiao Tong University, Shanghai, China	Wed 10:30, HS 4	p. 63
92	**Manuel Kauers** Johannes Kepler University, Linz, Austria		
93	**Abey Kelil** University of Pretoria, South Africa	Poster	p. 79
94	**Maurice Kenfack Nangho** University of Dschang, Cameroon	Tue 10:30, HS 4	p. 72
95	**Hironobu Kimura** Grad. school of Science and Technology, Kumamoto University, Japan	Tue 16:30, AM	p. 22
96	**Avner Kiro** Tel Aviv University, Israel	Tue 15:30, HS 5	p. 55
97	**Erik Koelink** Radboud University, Nijmegen, Netherlands	Thu 15:30, HS 6	p. 40
98	**Anna Kononova** Saint Petersburg State University, Russia	Thu 17:00, HS 4	p. 66
99	**Tom Koornwinder** University of Amsterdam, Netherlands	Thu 16:00, HS 6	p. 40
100	**Christoph Koutschan** RICAM, Austrian Academy of Sciences, Linz, Austria	Wed 11:00, HS 5	p. 50
101	**Christian Krattenthaler** Universität Wien, Vienna, Austria	Mon 09:00, AM	p. 13
102	**Arno Kuijlaars** Katholieke Universitait Leuven, Belgium	Wed 10:30, HS 3	p. 57
103	**Josef Küstner** Universität Wien, Vienna, Austria		
104	**Shara Lalo** Derby, UK	Mon 12:00, HS 6 Poster	p. 76 p. 79
105	**Zagros Lalo** Derby, UK		
106	**Imed Lamiri** Higher School of Sciences and Technology, Sousse University, Tunisia	Poster	p. 79
107	**Wayne Lawton** Siberian Federal University, Kransnoyarsk, Russia	Thu 16:00, HS 4	p. 65
108	**Marjolein Leurs** Katholieke Universitait Leuven, Belgium	Tue 10:30, HS 3	p. 45
109	**Chenkuan Li** Brandon University, Brandon, Manitoba, Canada	Wed 12:00, SH 02	p. 77
110	**Hélder Lima** University of Kent, Canterbury, UK	Mon 17:00, HS 3	p. 45
111	**Guillermo López Lagomasino** Universidad Carlos III de Madrid, Spain	Mon 10:30, HS 3	p. 43
112	**Abey López García** University of Central Florida, USA	Thu 15:30, HS 3	p. 59
113	**Ana Filipa Loureiro** University of Kent, Canterbury, UK	Tue 11:00, HS 3	p. 45

114	**Yu Luo** Graduate School of Informatics, Kyoto University, Japan	Thu 12:00, AM	p. 25
115	**Vladimir Lysov** Keldysh Institute of Applied Mathematics, Moscow, Russia	Thu 16:00, HS 3	p. 59
116	**Xinrong Ma** Department of Mathematics, Soochow University, Suzhou, China	Poster	p. 81
117	**Robert S. Maier** University of Colorado, Boulder, USA	Tue 11:00, AM	p. 32
118	**Manuel Mañas** Universidad Complutense de Madrid, Spain	Tue 10:30, HS 6	p. 35
119	**Juan F. Mañas-Mañas** Universidad de Almería, Spain	Tue 15:30, HS 6	p. 36
120	**Zeinab Mansour** Cairo University, Giza, Egypt	Mon 16:30, AM	p. 31
121	**Francisco Marcellán** Universidad Carlos III de Madrid, Spain	Mon 15:30, HS 6	p. 34
122	**Mariana Marčoková** University of Žilina, Slovakia		
123	**Clemens Markett** Aachen University of Technology, Germany	Mon 17:00, HS 6	p. 35
124	**Misael Marriaga** Universidad Rey Juan Carlos, Madrid, Spain	Wed 11:30, HS 6	p. 38
125	**Andrei Martínez Finkelshtein** Baylor University, Texas, USA	Mon 11:00, HS 3	p. 43
126	**Ana Matos** Laboratoire Paul Painlevé, Université de Lille, France	Thu 10:30, SH 02	p. 67
127	**Saiei-Jaeyeong Matsubara-Heo** Kobe University, Japan	Wed 11:30, AM	p. 23
128	**Dieudonne Mbouna** University of Coimbra, Portugal	Fri 11:30, HS 3	p. 62
129	**Sergio Medina Peralta** Universidad Carlos III de Madrid, Spain	Mon 12:00, HS 3	p. 44
130	**Marc Mezzarobba** Sorbonne Université, Campus Pierre et Marie Curie, Paris, France	Tue 10:30, HS 5	p. 48
131	**Johannes Middeke** RISC, Johannes Kepler University, Linz, Austria		
132	**Erwin Miña Díaz** University of Mississippi, USA	Thu 10:30, HS 3	p. 58
133	**Judit Mínguez** Universidad de La Rioja, Spain	Tue 11:30, HS 6	p. 35
134	**Leslie Molag** Katholieke Universitait Leuven, Belgium	Mon 11:00, AM Poster	p. 29 p. 80
135	**Luis Alejandro Molano Molano** Univ. Pedagógica y Tecnológica de Colombia, Duitama, Colombia	Tue 16:30, HS 6	p. 37
136	**Juan J. Moreno–Balcázar** Universidad de Almería, Spain	Mon 15:30, HS 6	p. 34

137	**Lucia Morey** National University of Córdoba, Argentina	Poster	p. 80
138	**Gargi Mukherjee** Johannes Kepler University, Linz, Austria		
139	**Hajime Nagoya** Kanazawa University, Ishikawa, Japan	Fri 11:30, AM	p. 28
140	**Caner Nazaroglu** Mathematical Institute, University of Cologne, Germany	Mon 11:00, HS 4	p. 70
141	**Gergő Nemes** Alfréd Rényi Institute of Mathematics, Budapest, Hungary	Tue 16:30, HS 5	p. 56
142	**Irina Nenciu** University of Illinois at Chicago, USA	Thu 14:00, AM	p. 19
143	**Thorsten Neuschel** Universität Bielefeld, Germany	Tue 12:00, HS 3	p. 46
144	**Geno Nikolov** Sofia University "St. Kliment Ohridski", Bulgaria	Thu 11:30, SH 02	p. 68
145	**Evans Doe Ocansey** RISC, Johannes Kepler University, Linz, Austria		
146	**Hiroshi Ogawara** Kumamoto University, Japan	Wed 11:00, AM	p. 23
147	**Adri B. Olde Daalhuis** University of Edinburgh, UK	Mon 10:30, AM Tue 16:00, HS 5	p. 29 p. 56
148	**Conor O'Rourke** Katholieke Universitait Leuven, Belgium		
149	**Peter Paule** RISC, Johannes Kepler University, Linz, Austria	Mon 17:00, HS 4	p. 72
150	**Henrik Laurberg Pedersen** University of Copenhagen, Denmark	Fri 10:30, AM	p. 27
151	**Marko Petkovšek** University of Ljubljana, Slovenia	Thu 11:30, HS 5	p. 51
152	**Hector Pijeira Cabrera** Universidad Carlos III de Madrid, Spain	Tue 12:00, HS 6	p. 36
153	**Veronika Pillwein** RISC, Johannes Kepler University, Linz, Austria	Tue 14:00, AM	p. 16
154	**Jiayue Qi** RISC, Johannes Kepler University, Linz, Austria		
155	**Clemens Raab** Johannes Kepler University, Linz, Austria		
156	**Silviu Radu** RISC, Johannes Kepler University, Linz, Austria	Tue 17:00, HS 4	p. 74
157	**Hjalmar Rosengren** Chalmers Univ. of Technology / University of Gothenburg, Sweden	Thu 17:00, HS 6	p. 41
158	**Margit Rösler** Paderborn University, Germany	Thu 16:30, HS 6	p. 41
159	**Diego Ruiz-Antolín** University of Cantabria, Santander, Spain	Poster	p. 80

160	**Edward Saff** Vanderbilt University, Nashville, Tennessee, USA		
161	**Klaus Schiefermayr** University of Applied Sciences Upper Austria, Wels, Austria	Wed 12:00, HS 4	p. 63
162	**Michael J. Schlosser** Universität Wien, Vienna, Austria	Tue 16:00, HS 3	p. 47
163	**Carsten Schneider** RISC, Johannes Kepler University, Linz, Austria	Mon 12:00, HS 4 Wed 11:30, HS 5	p. 71 p. 50
164	**Bastian Seifert** Universität Würzburg, Germany	Fri 10:30, HS 6	p. 41
165	**Diane Yahui Shi** Tianjin University, China		
166	**Genki Shibukawa** Kobe University, Japan	Fri 11:00, HS 6	p. 41
167	**Brian Simanek** Baylor University, Texas, USA	Thu 17:00, HS 3	p. 60
168	**Nicolas Smoot** RISC, Johannes Kepler University, Linz, Austria	Tue 16:00, HS 4	p. 73
169	**Mikhail Sodin** Tel Aviv University, Israel	Mon 14:00, AM	p. 14
170	**Alan Sokal** University College London, UK / New York University, USA	Tue 09:00, AM	p. 15
171	**Gopal Srinivasan** Indian Institute of Technology Bombay, India	Mon 10:30, HS 6	p. 75
172	**Tetiana Stepaniuk** RICAM, Austrian Academy of Sciences, Linz, Austria	Thu 17:00, HS 5	p. 69
173	**Marco Stevens** Katholieke Universitait Leuven, Belgium	Thu 16:30, AM	p. 26
174	**Armin Straub** University of South Alabama, USA	Thu 10:30, HS 5 Fri 09:00, AM	p. 51 p. 20
175	**Nikos Stylianopoulos** University of Cyprus, Cyprus	Thu 16:30, HS 5	p. 69
176	**Grzegorz Świderski** Mathematical Institute, University of Wrocław, Poland	Fri 10:30, HS 3	p. 60
177	**Franciszek Hugon Szafraniec** Jagiellonian University, Kraków, Poland	Tue 11:00, HS 6	p. 35
178	**Toshinori Takahashi** Kindai University, Japan	Mon 16:30, HS 5	p. 55
179	**Yumiko Takei** Graduate School of Science, Kobe University, Japan	Mon 15:30, HS 5	p. 54
180	**Daniel Duviol Tcheutia** Institute of Mathematics, University of Kassel, Germany	Fri 11:00, AM	p. 27
181	**Mikhail Tyaglov** Shanghai Jiao Tong University, China	Wed 11:30, SH 02	p. 77
182	**Ali Kemal Uncu** RISC, Johannes Kepler University, Linz, Austria	Tue 11:30, HS 4	p. 72

183	**Walter Van Assche** Katholieke Universitait Leuven, Belgium	Tue 15:30, HS 3	p. 46
184	**Wouter van de Vijver** Ghent University, Belgium	Fri 11:30, HS 6	p. 42
185	**Luis Verde-Star** Universidad Autónoma Metropolitana, Mexico City, Mexico	Tue 11:30, AM	p. 32
186	**Carlo Verschoor** University of Utrecht, Netherlands	Tue 12:00, AM	p. 33
187	**Luc Vinet** Centre de Recherches Mathématiques, Université de Montréal, Canada	Thu 09:00, AM	p. 18
188	**Lu Wei** University of Michigan, Dearborn, USA	Tue 11:00, HS 5	p. 48
189	**Aron Wennman** Tel Aviv University, Israel		
190	**Franck Wielonsky** CMI, Université Aix-Marseille, France	Thu 11:30, HS 3	p. 58
191	**Elaine Wong** RICAM, Austrian Academy of Sciences, Linz, Austria	Wed 10:30, HS 5	p. 49
192	**Yuan Xu** University of Oregon, USA	Thu 10:30, HS 6	p. 39
193	**Semyon Yakubovich** Faculdade da Ciências, Universidade do Porto, Portugal	Thu 10:30, AM	p. 25
194	**Luming Yao** City University of Hong Kong	Mon 12:00, HS 5	p. 54
195	**Maxim Yattselev** Indiana University-Purdue University, Indianapolis, USA	Mon 11:30, HS 3	p. 43
196	**Meesue Yoo** Dankook University, South Korea	Tue 17:00, HS 5	p. 76
197	**Peter Yuditskii** Johannes Kepler University, Linz, Austria	Thu 11:00, HS 4	p. 64
198	**Zafeirakis Zafeirakopoulos** Gebze Technology University, Turkey	Tue 11:00, HS 4	p. 72
199	**Ruiming Zhang** Northwest A&F University, Yangling, Shaanxi, China	Wed 11:00, SH 02	p. 77
200	**Maxim Zinchenko** University of New Mexico, Albuquerque, USA	Wed 11:00, HS 4	p. 63