Voros coefficients and the topological recursion for the hypergeometric differential equations associated with the 2-dimensional Garnier system

07.05 **Yumiko Takei**
(*Department of Mathematics, Graduate School of Science, Kobe University, Japan*)
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Abstract: The N-dimensional Garnier system is a Hamiltonian system with N independent variables obtained through monodromy preserving deformations of second order linear differential equations on $\mathbb{P}^1$ with $N+3$ regular singular points. In the case of $N=1$, the system reduces to the sixth Painlevé equation P_{VI} and the Gauss hypergeometric function gives a particular solution of P_{VI}. Note that each member of the family of the Gauss hypergeometric equations is obtained from P_{VI} by the so-called confluence process. In the same manner the hypergeometric differential equations with two independent variables are obtained from the 2-dimensional Garnier system. In this talk I consider some relationship between the exact WKB analysis and the topological recursion for the hypergeometric differential equations with two independent variables.

Exact WKB analysis is a powerful tool to study differential equations globally. In particular, Voros coefficients provide important quantities for describing global behavior of solutions of differential equations. On the other hand, the topological recursion introduced by B. Eynard and N. Orantin [2] to study the correlation functions in the random matrix theory gives a generalization of the loop equations for the matrix model. Recently, several surprising connections between exact WKB analysis and topological recursion have been discovered. For example, it is shown that WKB solutions are constructed via the topological recursion [1]. Furthermore, together with Iwaki and Koike I show that in the case of the family of the Gauss hypergeometric equations Voros coefficients are described by the generating functions of free energies defined in terms of the topological recursion [3,4].

In this talk, I would like to discuss a generalization of the above result, that is, I will report that the Voros coefficients for some confluent hypergeometric differential equations of two variables associated with degenerate 2-dimensional Garnier systems are described by the generating functions of free energies defined in terms of the topological recursion. As its application I will also show that the following objects can be computed in an explicit manner: (i) three-term difference equations that the generating function of the free energies satisfies, (ii) explicit form of free energies, and (iii) explicit form of Voros coefficients.

[1] V. Bouchard and B. Eyanard, Reconstructing WKB from topological recursion, *Journal de l'Ecole polytechnique – Mathematiques*, **4** (2017), pp. 845–908.
[2] B. Eynard and N. Orantin, Invariants of algebraic curves and topological expansion, *Communications in Number Theory and Physics*, **1** (2007), pp. 347–452; arXiv:math-ph/0702045.
[3] K. Iwaki, T. Koike, and Y.-M. Takei, Voros coefficients for the hypergeometric differential equations and Eynard–Orantin's topological recursion, part I: for the Weber equation; arXiv:1805.10945.
[4] K. Iwaki, T. Koike, and Y.-M. Takei, Voros coefficients for the hypergeometric differential equations and Eynard–Orantin's topological recursion, part II: for the confluent family of hypergeometric equations, preprint; arXiv:1810.02946.