Asymptotics of orthogonal polynomials with unbounded recurrence coefficients

08.13 **Grzegorz Świderski**
(*Mathematical Institute, University of Wrocław, Poland*)
Time: Friday 26.07., 10:30 - 11:00, Room HS 3

Abstract: Let μ be a probability measure on the real line with all moments finite. Let $(p_n : n \geq 0)$ be the corresponding sequence of orthonormal polynomials. It satisfies

$$\begin{gathered} p_0(x) = 1, \quad p_1(x) = \frac{x - b_0}{a_0}, \\ a_{n-1}p_{n-1}(x) + b_n p_n(x) + a_n p_{n+1}(x) = x p_n(x) \quad (n \geq 1), \end{gathered} \tag{1}$$

for some sequences $a_n > 0$ and $b_n \in \mathbb{R}$. Conversely, Favard's theorem states that every sequence of polynomials satisfying (**??**) is orthonormal with respect to some measure μ. The measure μ is unique if the Carleman condition is satisfied, i.e. when

$$\sum_{k=0}^{\infty} \frac{1}{a_k} = \infty.$$

In the proposed talk we are interested in the asymptotic behaviour of orthogonal polynomials in terms of its recurrence coefficients. Let $N \geq 1$ be an integer. In the analysis the crucial role is played by the so-called N-step transfer matrix defined by

$$X_n(x) = \prod_{j=n}^{n+N-1} \begin{pmatrix} 0 & 1 \\ -\frac{a_{j-1}}{a_j} & \frac{x-b_j}{a_j} \end{pmatrix}.$$

We are going to present the following

Theorem 1. *Let $N \geq 1$ be a positive integer and let $i \in \{0, 1, \ldots, N-1\}$. Suppose that the sequence $(X_{nN+i} : n \in \mathbb{N})$ is of bounded variation and let $\mathcal{X}_i$ be its limit. Let*

$$\Lambda = \{x \in \mathbb{R} : |\operatorname{tr} \mathcal{X}_i(x)| < 2\}.$$

If $\det \mathcal{X}_i = 1$ and the Carleman condition is satisfied, then the measure μ is purely absolutely continuous on Λ and there is a continuous real-valued function η such that

$$\sqrt{a_{kN+i-1}}\, p_{kN+i}(x) = \sqrt{\frac{2|[\mathcal{X}_i(x)]_{21}|}{\pi \mu'(x) \sqrt{4 - (\operatorname{tr} \mathcal{X}_i(x))^2}}} \sin\Big(\sum_{j=1}^{k} \theta_j(x) + \eta(x)\Big) + \epsilon_k(x), \qquad x \in \Lambda$$

for explicit θ_j and a constructive upper bound on ϵ_k.

Theorem 1 is an extension of results obtained in Máté-Nevai-Totik (1985), Geronimo-Van Assche (1991) and Aptekarev-Geronimo (2016). Our approach is based on uniform diagonalisation of transfer matrices.

We are going to present the applications of Theorem 1 with unbounded a_n to the asymptotics of Christoffel functions with the rate of convergence and to universality limits of Christoffel-Darboux kernel.

This is a joint work with Bartosz Trojan (Polish Academy of Sciences).