## Chebyshev polynomials on circular arcs

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**Time:** Wednesday 24.07., 12:00 - 12:30, Room HS 4

**Abstract:** In this talk, we give an explicit representation of the Chebyshev polynomial on a given circular arc

$$A_\alpha := \big\{z \in \mathbb{C} : |z| = 1, -\alpha \le \arg(z) \le \alpha\big\}, \qquad 0 < \alpha \le \pi,$$

(a problem which was first considered in [1]), which is done in two steps: In the first step, following [2], we give an explicit representation of the Chebyshev polynomial (of degree $N$) on $A_\alpha$ in terms of the Chebyshev polynomial with respect to the weight function $w(x) := 1$ (for $N$ even) and $w(x) := \sqrt{1-x^2}$ (for $N$ odd) on the two real intervals $[-1,-a] \cup [a,1]$, where $a := \cos(\frac{\alpha}{2})$. For this representation, we will need the mapping $z \mapsto \frac{1}{2}\Big(\sqrt{z} + \frac{1}{\sqrt{z}}\Big)$ which maps $\{z \in \mathbb{C} : |z| = 1,\ \mathrm{Im}\{z\} \ge 0\}$ bijectively onto the interval $[0,1]$. In the second step, these Chebyshev polynomials (with respect to $w(x) := 1$ and $w(x) := \sqrt{1-x^2}$) are represented with the help of Jacobian elliptic and theta functions. These representations go back to [3] and [4]. The talk is based on the paper [5].

[1] J.-P. Thiran and C. Detaille, *Chebyshev polynomials on circular arcs in the complex plane*, Progress in approximation theory, Academic Press, 1991, pp. 771–786.

[2] F. Peherstorfer and K. Schiefermayr, *On the connection between minimal polynomials on arcs and on intervals*, in "Functions, Series, Operators" (Budapest, 1999), János Bolyai Math. Soc., Budapest, 2002, pp. 339–356.

[3] N.I. Akhiezer, *Über einige Funktionen, die in gegebenen Intervallen am wenigsten von Null abweichen*, Bull. Soc. Phys.-Math. Kazan, III. Ser. **3** (1928), 1–69 (in German).

[4] E.I. Krupickiĭ, *On a class of polynomials with least deviation from zero on two intervals*, Dokl. Akad. Nauk SSSR **138** (1961), 533–536.

[5] K. Schiefermayr, *Chebyshev polynomials on circular arcs*, to appear in Acta Sci. Math. (Szeged).