New bounds for the extreme zeros of classical orthogonal polynomials

10.03 **Geno Nikolov**
(*Sofia University "St. Kliment Ohridski", Bulgaria*)
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Abstract: The zeros of classical orthogonal polynomials have been a topic of intensive investigation. There are many reasons for this interest, such as the nice electrostatic interpretation of the zeros of the Jacobi, Laguerre and Hermite polynomials, their important role as nodes of Gaussian quadrature formulae, as well as the key role these zeros play in the proofs of some classical inequalities.

Derivation of sharp upper and lower bounds for the extreme zeros is of particular interest. For this, powerful analytic and discrete techniques have been developed. Among them are Sturms comparison theorem for the zeros of solutions of second order differential equations, A. Markovs theorem on monotonicity of zeros of orthogonal polynomials in terms of the behavior of the weight function, the Hellmann-Feynman theorem on variation of eigenvalues of Hermitian matrices, the Obrechkoff theorem on Descartess rule of signs, the Wall-Wetzel theorem on eigenvalues of Jacobi matrices in terms of chain sequences, etc.

We apply the Euler-Rayleigh method to obtain some new bounds for the extreme zeros of the Jacobi (in particular, of Gegenbauer) and Laguerre polynomials. Typically, the comparison of the different estimates does not single out a "best" bound as these estimates depend on two or three parameters. We show that our bounds improve some of the best known bounds obtained recently.