On orthogonal polynomials and the long-time behavior of completely integrable systems

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Abstract: The study of any differential equation aims to reveal, in as much detail as possible, the behavior of solutions especially as time goes to infinity. While this is in most cases a lofty goal, it becomes much more approachable for completely integrable equations. These are, loosely speaking, nonlinear differential equations with a sufficiently large number of conserved quantities, and in their simplest, finite dimensional incarnation they are subject to the Liouville-Arnold-Jost (LAJ) theorem, which guarantees the existence of a global change of variables linearizing the flow. Thus completely integrable equations are much more approachable mathematically than non-integrable ones, while still exhibiting a much richer range of behaviors than linear differential equations do.

The catch in the program sketched above is that the LAJ Theorem, while absolutely remarkable, is not constructive. But in order to fully exploit the completely integrable structure of a differential equation, one needs to find the specific change of variables which realizes the linearization. The goal of my talk is three-fold. First, I will sketch the general theory of complete integrability. I will then focus on two concrete completely integrable partial difference equations, namely the Toda lattice and the (defocusing) Ablowitz-Ladik equations. In what is known as the finite case, finding the linearizing change of variables for both of these equations relies in a fundamental way on the theory of orthogonal polynomials on the real line and the unit circle, and I will spend most of the talk on clarifying this connection. Finally, I will consider these same equations in the periodic setting, where more subtle aspects of orthogonal polynomial theory are needed. Time permitting, I will conclude with some recent work and open questions related to the analysis of these cases.