On the Borel summability of WKB solutions near a simple pole

07.10 Gergő Nemes

(Alfréd Rényi Institute of Mathematics, Budapest, Hungary) **Time:** Tuesday 23.07., 16:30 - 17:00, Room HS 5

Abstract: We consider the following Schrödinger-type differential equation:

$$\frac{\mathrm{d}^2 W(u,\xi)}{\mathrm{d}\xi^2} = (u^2 + \psi(\xi))W(u,\xi),$$

where u is a large positive parameter and $\psi(\xi)$ is an analytic function of ξ apart from countably many singularities. It is known that this equation has formal solutions of the form

$$W_{1,2}(u,\xi) = e^{\pm\xi u} \sum_{n=0}^{\infty} \frac{A_n(\pm\xi)}{u^n}.$$

These are called the WKB solutions. We study the Borel summability of these WKB solutions near a simple pole of the function $\psi(\xi)$ which we assume to be located at the origin. It is shown that both of the formal series are Borel summable in every closed strip $\{\xi : |\Im\xi| \leq \gamma\}$ contained in the domain of analyticity of $\psi(\xi)$ apart from the Stokes rays $\arg \xi = 0$, 2π and $\arg \xi = \pm \pi$ emanating from the origin. We determine the type of singularities of the Borel transforms near the origin when $|\xi|$ is small and also provide global connection formulae between the solutions $W_1(u,\xi)$ and $W_2(u,\xi)$.