MS11: Developments in q-series and the theory of partitions

Organizer: Ali Uncu (*Johannes Kepler University Linz, Austria*)

This mini-symposium is dedicated to discuss recent developments in the study of q-series and its implications on the theory of partitions in a broad perspective. We aim to welcome the representation of all the techniques used in the field such as series manipulations, basic hypergeometric transformations, modular forms, bijective combinatorics, etc.

New representations for $\sigma(q)$ via reciprocity theorems

11.01 **Koustav Banerjee**
(*Research Institute for Symbolic Computation, Johannes Kepler University, Linz, Austria*)
Time: Monday 22.07., 10:30 - 11:00, Room HS 4

Abstract: In this talk, we will see two new representations for Ramanujan's function $\sigma(q)$. The proof of the first one uses the three-variable reciprocity theorem due to Soon-Yi Kang and a transformation due to R.P. Agarwal while that of the second uses the four-variable reciprocity theorem due to George Andrews and a generalization of a recent transformation of Andrews, Schultz, Yee and the second author. The advantage of these representations is that they involve free complex parameters - one in the first representation, and two in the second. The partition theoretic interpretation has been already made in the context of three variable representation by Atul Dixit and Bibekananda Maji.

This is a joint work with Prof. Atul Dixit.

Modular properties of false theta functions

11.02 **Caner Nazaroglu**
(*Mathematical Institute, University of Cologne, Germany*)
Time: Monday 22.07., 11:00 - 11:30, Room HS 4

Abstract: False theta functions are functions that closely resemble classical theta functions, which despite this similarity do not have the modular properties that theta functions possess. They appear, for example, in the context of link invariants, W-algebras and also are closely related to mock modular forms. In this talk, I will describe modular properties of false theta functions and give a modular completion analogous to modular completions of mock modular forms. Finally, I will give an application of this machinery to derive a Rademacher type exact expression for the number of unimodal sequences and extend earlier work on their asymptotic properties.

New results on asymptotics and inequalities for partition functions

11.03 **Alexandru Ciolan**
(*Mathematical Institute, University of Cologne, Germany*)
Time: Monday 22.07., 11:30 - 12:00, Room HS 4

Abstract: We present some recent results on asymptotics and inequalities for (over)partitions. In the first part of this talk we prove a conjecture by Bringmann and Mahlburg (2012), which says that a large enough number n has more partitions into squares with an even number of parts than with an odd number of parts if n is even, and conversely if n is odd. In the second part we compute asymptotics for $\overline{N}(a, c, n)$, the number of overpartitions of n with a number of parts congruent to a modulo c and use these asymptotics to prove some inequalities on overpartition ranks conjectured by Ji, Zhang and Zhao (2018), and Wei and Zhang (2018).

Symbolic Summation, difference ring algorithms and q-applications

11.04

Carsten Schneider
(*Research Institute for Symbolic Computation, Johannes Kepler University, Linz, Austria*)
Time: Monday 22.07., 12:00 - 12:30, Room HS 4

Abstract: In the last years the difference ring approach for symbolic summation has been pushed forward substantially for multiple sums defined over hypergeometric products. Due to its generality, this approach also works for the q-hypergeometric, multibasic or mixed multibasic case. But so far it has been neglected up to some toy examples. In this talk I will report on new developments in cooperation with Jakob Ablinger and Ali K. Uncu to push forward the difference ring machinery for the q-world. Examples will be presented in which, e.g., q-trinomial coefficients arise.

Enumerating simultaneous core partitions into k distinct parts

11.05

Hannah Burson
(*University of Illinois at Urbana-Champaign, USA*)
Time: Monday 22.07., 15:30 - 16:00, Room HS 4

Abstract: In 2016, using a combinatorial bijection with certain abaci diagrams, Nath and Sellers enumerated $(s, ms \pm 1)$-core partitions into distinct parts. In this talk, we explain new generalizations of this theorem, with a focus on a generating polynomial that enumerates simultaneous core partitions by the number of parts.

This work is joint with Simone Sisneros-Thiry and Armin Straub.

On a continued fraction of Ramanujan

11.06

Gaurav Bhatnagar
(*University of Vienna, Austria*)
Time: Monday 22.07., 16:00 - 16:30, Room HS 4

Abstract: We study a continued fraction due to Ramanujan, that he recorded as Entry 12 in Chapter 16 of his second notebook. It is presented in Part III of Berndt's volumes on Ramanujan's notebooks. We give two alternate approaches to proving Ramanujan's Entry 12, one using a method of Euler, and another using the theory of orthogonal polynomials. We consider a natural generalization of Entry 12 suggested by the theory of orthogonal polynomials.

This is joint work with Mourad Ismail.

Some q-series conjectures related to Rogers-Ramanujan type identities of Kanade and Russell

11.07

Chris Jennings-Shaffer
(*Mathematical Institute, University of Cologne, Germany*)
Time: Monday 22.07., 16:30 - 17:00, Room HS 4

Abstract: In 2014, and in follow-up work in 2018, Kanade and Russell conjectured a large number of identities of the Rogers-Ramanujan type. Recently, in joint work with Kathrin Bringmann and Karl Mahlburg, we proved seven of their conjectures. Rather than focus on the proven identities, here we discuss some conjectural identities for certain q-hypergeometric series that came out of a reduction of other conjectures of Kanade and Russell. In these cases, the series can be expressed as ${}_2\phi_1$ series, rather than multi-sums, and the result should be a simple infinite product. While this sounds quite simple, somehow a full proof has eluded us.

Modular functions and computer algebra

11.08 **Peter Paule**
(*Research Institute for Symbolic Computation, Johannes Kepler University, Linz, Austria*)
Time: Monday 22.07., 17:00 - 17:30, Room HS 4

Abstract: The talk reports on recent progress concerning computer algebra related to modular functions. Examples are a proof of the Weierstrass gap theorem without using the Riemann-Roch formula, or a new algorithmic framework for proving Ramanujan's celebrated congruences for partition numbers.

Most of the results presented arose in joint work with Silviu Radu (RISC).

Exponential function on nonuniform lattices and solutions to some q-indeterminate moment problems

11.09 **Maurice Kenfack Nangho**
(*Department of Mathematics and Computer Science, University of Dschang, Cameroon*)
Time: Tuesday 23.07., 10:30 - 11:00, Room HS 4

Abstract: We develop analogs of exponential and trigonometric functions (including the basic exponential function) and derive their fundamental properties: addition formula, positivity, reciprocal and fundamental relations of trigonometry. We establish a binomial theorem provide a formula for computing the nth-derivatives for analytic functions on nonuniform lattices (q-quadratic and quadratic variables). We also develop solutions to q-indeterminate moment problems related with Askey-Wilson polynomials. This talk is based on [1,2].

[1] M. Kenfack Nangho, M. Foupouagnigni and W. Koepf: On exponential and trigonometric functions on nonuniform lattices, Ramanujan Journal, Volume 49, Issue 1, May 2019, Pages 1–37.
[2] M. Kenfack Nangho and K. Jordaan: A characterization of Askey-Wilson polynomials, Proceedings of the AMS, Volume 147, Number 6, June 2019, Pages 2465–2480.

Using geometry for computing q-series

11.10 **Zafeirakis Zafeirakopoulos**
(*Gebze Technology University, Turkey*)
Time: Tuesday 23.07., 11:00 - 11:30, Room HS 4

Abstract: Computing generating functions of objects defined by linear constraints (e.g., the majority of integer partition families) is an old but interesting game. Many counting generating functions have nice representations as q-series and all kinds of amazing identities or relations emerge. On the other hand, the full (multivariate) generating function contains more information, but it rarely gives rise to nice relations. Using polyhedral geometry, we can express the rational generating function for a set of combinatorial objects (e.g., solid partitions on a cube) as a sum of symbolic cones. Symbolic cones are a "multivariate" representation that does not suffer from the usual combinatorial explosion during computation, thus can be used for computing q-series.

In this talk, we will present the notion of symbolic cone, why it is computationally and conceptually preferable over rational functions and how to obtain q-series using Polyhedral Omega.

Multisums related to Rogers–Ramanujan type identities and their refinements

11.11 **Ali Kemal Uncu**
(*Research Institute for Symbolic Computation, Johannes Kepler University, Linz, Austria*)
Time: Tuesday 23.07., 11:30 - 12:00, Room HS 4

Abstract: There has been a recent influx of Andrews–Gordon identities like multi nested q-series representations of the generating functions for the number of partitions with certain difference conditions. We

will present how one can refine these generating functions by imposing bounds on the largest part of the related partitions. These refinements will lead to many polynomial identities.

The Deng-Yang conjecture

11.12 **Mourad E. H. Ismail**
(*University of Central Florida, USA*)
Time: Tuesday 23.07., 12:00 - 12:30, Room HS 4

Abstract: Ding and Yang conjectured that the q-normal density (the weight function for q-Hermite polynomials suitably normalized) tends to a sum of two delta functions. We prove their conjecture and show that this a general property of most of the q-orthogonal polynomials. This is based on joint work with D. Dai and X. W. Wang.

Ruscheweyh-type starlike functions of complex order associated with q-difference operator

11.13 **Asena Çetinkaya**
(*Istanbul, Turkey*)
Time: Tuesday 23.07., 15:30 - 16:00, Room HS 4

Abstract: Quantum calculus or q-calculus dates back to Leonhard Euler (1707–1783) and Carl Gustav Jacobi (1804–1851). But q-calculus became popular only after its usefulness in quantum mechanics after 1905 paper by Albert Einstein. In 1909 and 1910 Jackson initiated in-depth study of q-calculus. He was the first to develop the q-integral and q-derivative in a systematic way. The great interest is due to its applications in various branches of mathematics and physics, as for example, in the areas of ordinary fractional calculus, orthogonal polynomials, basic hypergeometric functions, combinatorics.

In this paper, we investigate three new subclasses of Ruscheweyh-type starlike functions of complex order associated with q- difference operator. We investigate inclusion theorem, sufficient coefficient estimates, distortion bounds and radius of starlikeness of these subclasses. Further, we obtain partial sums of our classes.

Some new Ramanujan–Kohlberg identities

11.14 **Nicolas Smoot**
(*Research Institute for Symbolic Computation, Johannes Kepler University, Linz, Austria*)
Time: Tuesday 23.07., 16:00 - 16:30, Room HS 4

Abstract: Ramanujan's identities involving the generating functions for $p(5n + 4)$ and $p(7n + 5)$ are considered to be among his most beautiful results. These were shown by Kolberg to be special cases of a larger class of relationships expressing generating functions for $p(mn + j)$ in terms of eta quotients. The form of these identities is prevalent throughout the theory of partitions. They are useful in the verification of families of partition congruences, as well as in the study of certain conjectures in the theory of modular functions. In 2014 Silviu Radu developed an algorithm to compute the Ramanujan-Kolberg identities inherent in various arithmetic functions. We have fully implementated this algorithm with Mathematica. We will show some interesting examples found using our implementation, with a focus on partition congruences. We include some new results, as well as some interesting improvements on previous results.

A q-analogue for Euler's $\zeta(6) = \pi^6/945$

11.15 **Ankush Goswami**
(*University of Florida, USA / RISC, Johannes Kepler University, Linz, Austria*)
Time: Tuesday 23.07., 16:30 - 17:00, Room HS 4

Abstract: Recently, Z.-W. Sun obtained q-analogues of Euler's formula for $\zeta(2)$ and $\zeta(4)$. Sun's formula

were based on identities satisfied by triangular numbers and properties of Euler's q-Gamma function. In this talk, we discuss a q-analogue of $\zeta(6) = \pi^6/945$. Indeed, we have been able to obtain q-analogues of Euler's formula for $\zeta(2k)$, $k = 4, 5, \ldots$ (the general case). However, it is to be noted here that the case $k = 3$ or the q-analogue of $\zeta(6)$ is striking as it leads to very interesting connections. Also, the q-analogue of $\zeta(6)$ is the first non-trivial case where we see the occurence of a certain "extra" term which goes to zero as $q \to 1$ from inside the unit disk. We will also shed some light on this extra term.

Proofs of some q-product identities conjectured by Merca

11.16

Silviu Radu
(*Research Institute for Symbolic Computation, Johannes Kepler University, Linz, Austria*)
Time: Tuesday 23.07., 17:00 - 17:30, Room HS 4

Abstract: In the present talk we show how to prove several infinite q-product identities using modular functions. These q-product identities have been conjectured by Mircea Merca. This is joint work with Christian Krattenthaler and Mircea Merca.