MS09: Extremal polynomials and almost periodicity

Organizers: Jacob S. Christiansen (*Lund University, Sweden*)
Benjamin Eichinger (*Rice University, Houston, USA*)
Tom VandenBoom (*Yale University, New Haven, USA*)

In this mini-symposium we aim to discuss recent advances in the field of extremal polynomials, such as orthogonal and Chebyshev polynomials. New developments for the associated operators (given by, e.g., Jacobi or CMV matrices) as well as for continuous Schrödinger operators and canonical systems are naturally included. We are particularly interested in situations where almost periodicity occurs.

Bernstein-type inequalities and geometric function theory

09.01 **Sergei Kalmykov**
(*Shanghai Jiao Tong University, Shanghai, China*)
Time: Wednesday 24.07., 10:30 - 11:00, Room HS 4

Abstract: We will mainly discuss results concerning Bernstein- and Markov-type inequalities for polynomials and rational functions with restriction on zeros or curved majorants on the interval $[-1, 1]$ or an arc. Methods of proofs are based on applying geometric function theory of complex variables. Special attention will be paid to cases of equality, construction of extremal polynomials and rational functions and mapping properties of these functions.

Lower bounds for extremal polynomials

09.02 **Maxim Zinchenko**
(*University of New Mexico, Albuquerque, USA*)
Time: Wednesday 24.07., 11:00 - 11:30, Room HS 4

Abstract: In this talk I will discuss lower bounds on the norms of L_p extremal polynomials. This work is an attempt to find sharp lower bounds that parallel the lower bounds for Chebyshev polynomials.

Periodic ellipsoidal billiards and extremal polynomials

09.03 **Vladimir Dragovic**
(*University of Texas at Dallas, USA*)
Time: Wednesday 24.07., 11:30 - 12:00, Room HS 4

Abstract: A comprehensive study of periodic trajectories of the billiards within ellipsoids in the d-dimensional Euclidean space is presented. The novelty of the approach is based on a relationship established between the periodic billiard trajectories and the extremal polynomials of the Chebyshev type on the systems of d intervals on the real line. The case study of trajectories of small periods T,$d \leq T \leq 2d$ is given. In particular, it is proven that all d-periodic trajectories are contained in a coordinate-hyperplane and that for a given ellipsoid, there is a unique set of caustics which generates $d+1$-periodic trajectories. A complete catalog of billiard trajectories with small periods is provided for $d = 2$ and 3. This is a joint work with Milena Radnovic.

Chebyshev polynomials on circular arcs

09.04 **Klaus Schiefermayr**
(*University of Applied Sciences Upper Austria, Wels, Austria*)
Time: Wednesday 24.07., 12:00 - 12:30, Room HS 4

Abstract: In this talk, we give an explicit representation of the Chebyshev polynomial on a given circular arc

$$A_\alpha := \{z \in \mathbb{C} : |z| = 1, -\alpha \leq \arg(z) \leq \alpha\}, \qquad 0 < \alpha \leq \pi,$$

(a problem which was first considered in [1]), which is done in two steps: In the first step, following [2], we give an explicit representation of the Chebyshev polynomial (of degree N) on A_α in terms of the Chebyshev polynomial with respect to the weight function $w(x) := 1$ (for N even) and $w(x) := \sqrt{1-x^2}$ (for N odd) on the two real intervals $[-1,-a] \cup [a,1]$, where $a := \cos(\frac{\alpha}{2})$. For this representation, we will need the mapping $z \mapsto \frac{1}{2}\left(\sqrt{z} + \frac{1}{\sqrt{z}}\right)$ which maps $\{z \in \mathbb{C} : |z| = 1,\ \mathrm{Im}\{z\} \geq 0\}$ bijectively onto the interval $[0,1]$. In the second step, these Chebyshev polynomials (with respect to $w(x) := 1$ and $w(x) := \sqrt{1-x^2}$) are represented with the help of Jacobian elliptic and theta functions. These representations go back to [3] and [4]. The talk is based on the paper [5].

[1] J.-P. Thiran and C. Detaille, *Chebyshev polynomials on circular arcs in the complex plane*, Progress in approximation theory, Academic Press, 1991, pp. 771–786.

[2] F. Peherstorfer and K. Schiefermayr, *On the connection between minimal polynomials on arcs and on intervals*, in "Functions, Series, Operators" (Budapest, 1999), János Bolyai Math. Soc., Budapest, 2002, pp. 339–356.

[3] N.I. Akhiezer, *Über einige Funktionen, die in gegebenen Intervallen am wenigsten von Null abweichen*, Bull. Soc. Phys.-Math. Kazan, III. Ser. **3** (1928), 1–69 (in German).

[4] E.I. Krupickiĭ, *On a class of polynomials with least deviation from zero on two intervals*, Dokl. Akad. Nauk SSSR **138** (1961), 533–536.

[5] K. Schiefermayr, *Chebyshev polynomials on circular arcs*, to appear in Acta Sci. Math. (Szeged).

Continued fraction expansions and generalized indefinite strings

09.05

Jonathan Eckhardt
(*Loughborough University, UK*)

Time: Thursday 25.07., 10:30 - 11:00, Room HS 4

Abstract: Stieltjes continued fraction expansions play a decisive role in the solution of the inverse spectral problem for Krein strings. Certain continued fractions of a modified form correspond in the same way to generalized indefinite strings. I will discuss under which conditions Herglotz-Nevanlinna functions allow such an expansion and use this to solve the inverse spectral problem for generalized indefinite strings with coefficients supported on a discrete set. These results are related to the Hamburger moment problem as well as multi-soliton solutions of particular integrable wave equations.

Martin functions of Fuchsian groups and character automorphic subspaces of the Hardy space in the upper half plane

09.06

Peter Yuditskii
(*Johannes Kepler University, Linz, Austria*)

Time: Thursday 25.07., 11:00 - 11:30, Room HS 4

Abstract: We establish exact conditions for non triviality of all subspaces of the standard Hardy space in the upper half plane, that consist of the character automorphic functions with respect to the action of a discrete subgroup of $SL_2(\mathbb{R})$. Such spaces are the natural objects in the context of the spectral theory of almost periodic differential operators and in the asymptotics of the approximations by entire functions. A naive idea: it should be completely parallel to the celebrated Widom characterization for Hardy spaces on Riemann surfaces with a minor modification, namely, one has to substitute the Green function of the domain with the Martin function. Basically, this is correct, but...

The talk is based on a joint work with Aleksandr Kheifets.

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Szegő condition and scattering for Dirac operators

09.07

Roman Bessonov
(*Saint Petersburg State University and PDMI, Russia*)

Time: Thursday 25.07., 11:30 - 12:00, Room HS 4

Abstract: A classical fact of scattering theory for one-dimensional Dirac operator is the existence of

strong wave operators in the case where the potential of the operator is absolutely integrable over the real line. Much more deep results by Christ, Kiselev (2002) and Denisov (2004) establish the existence of wave operators for potentials in L^p for $1 < p < 2$ and $p = 2$, correspondingly. In all cases the spectral measure of the Dirac operator under consideration belongs to the Szegő class on the real line: its density coincides with the absolute value of an outer function. We show that this condition always imply existence of wave operators, which allows us to describe a broad class of "large" potentials for which the wave operators exist.

Schrödinger operators with substitutive potentials beyond linear complexity

09.08

Philipp Gohlke
(*Universität Bielefeld, Germany*)
Time: Thursday 25.07., 12:00 - 12:30, Room HS 4

Abstract: Tight-binding Schrödinger operators with potentials generated by primitive substitutions have been studied extensively in the past. Typically, these models exhibit singular continuous spectra of Lebesgue measure zero. We consider classes of substitutional systems that go beyond both minimality and linear complexity and explore some of the novel spectral phenomena that can occur. This is joint work with B. Eichinger.

Periodic coordinates and a magic formula for finite-gap CMV matrices

09.09

Benjamin Eichinger
(*Rice University Houston, Texas, USA*)
Time: Thursday 25.07., 15:30 - 16:00, Room HS 4

Abstract: We prove a bijective unitary correspondence between 1) the isospectral torus of almost-periodic, absolutely continuous CMV matrices having fixed finite-gap spectrum E and 2) periodic block-CMV matrices satisfying a *Magic Formula*. This latter class arises as E-dependent operator Möbius transforms of certain generating CMV matrices which are periodic up to a rotational phase; for this reason we call them "MCMV". Naturally, this has also consequences for the associated Schur functions. We show that for any Schur function associated to a finite-gap CMV matrix (and therefore with almost periodic Verblunsky coefficients) there exists a more general Nevanlinna-Pick interpolation problem with periodic interpolation data.

The talk is based on a joint work with J. S. Christiansen and T. VandenBoom.

Values distribution of almost periodic functions, spectral factorization and entire functions

09.10

Wayne Lawton
(*Siberian Federal University, Kransnoyarsk, Russia*)
Time: Thursday 25.07., 16:00 - 16:30, Room HS 4

Abstract: We recently derived an upper bound on the mean measure of the set where a nonzero Bohr almost periodic function f with bounded spectra has small modulus. This bound implies that $\log|f|$ is almost periodic in the sense of Besicovitch. We use this result to relate the Ahiezer factorization of an entire extension F of a nonnegative Bohr almost periodic function f with a bounded spectrum and the Helson-Lowdenslager spectral factorizations of its lift h to a compact group. We also discuss new classes of one dimensional quasicrystals related to both Bohr and Besicovitch almost periodic functions.

CMV block matrices for symmetric matrix measures on the unit circle

09.11

Luis E. Garza
(*Facultad de Ciencias, Universidad de Colima, Mexico*)
Time: Thursday 25.07., 16:30 - 17:00, Room HS 4

Abstract: Abstract: In this contribution, we study the relationship between the CMV block matrices associated with two symmetric positive definite matrix measures supported on the unit circle. We also consider certain transformations of an orthogonality matrix measure supported on the unit circle, and deduce connection formulas for the corresponding orthogonal families and their Verblunsky matrix coefficients.

Szegő minimum problem and Nevai's conjecture

09.12

Anna Kononova
(*Saint Petersburg State University, Russia*)
Time: Thursday 25.07., 17:00 - 17:30, Room HS 4

Abstract: We will present several quantitative results concerning with the Szegő minimum problem for classes of measure on the unit circle with divergent logarithmic integral. In particular, we refute a known Nevai's conjecture.

The talk is based on a joint work with A. Borichev and M. Sodin (arXiv:1902.00874, arXiv:1902.00872)