MS05: Multiple orthogonal polynomials and Hermite-Padé approximation

Organizer: Walter van Assche (*Katholieke Universiteit Leuven, Belgium*)

Multiple orthogonal polynomials are polynomials in one variable that satisfy orthogonality relations with respect to r measures. They appear in a natural way in Hermite-Padé approximation, which is simultaneous rational approximation to r functions near infinity. The session will focus on special families of multiple orthogonal polynomials, asymptotic behavior of the zeros, asymptotic results and Riemann-Hilbert/steepest descent, applications in numerical analysis, random matrices, determinantal point processes.

Asymptotics of Cauchy biorthogonal polynomials

05.01 **Guillermo López Lagomasino**
(*Universidad Carlos III de Madrid, Spain*)
Time: Monday 22.07., 10:30 - 11:00, Room HS 3

Abstract: We consider sequences of biorthogonal polynomials with respect to a Cauchy type convolution kernel and give the weak and ratio asymptotic of the corresponding sequences of biorthogonal polynomials. The construction is intimately related with a mixed type Hermite-Padé approximation problem whose asymptotic properties is also revealed.

This is joint work with Sergio Medina Peralta and Ulises Fidalgo

Asymptotics of multiple orthogonal polynomials for cubic weight

05.02 **Andrei Martínez Finkelshtein**
(*Baylor University, Texas, USA*)
Time: Monday 22.07., 11:00 - 11:30, Room HS 3

Abstract: We consider the type I and type II multiple orthogonal polynomials (MOPs), satisfying non-hermitian orthogonality with respect to the weight $\exp(-z^3)$ on two unbounded contours on the complex plane. Under the assumption that the orthogonality conditions are distributed with a fixed proportion α, we ?nd the detailed (rescaled) asymptotics of these MOPs, and describe the phase transitions of this limit behavior as a function of α. This description is given in terms of the vector critical measure, the saddle point of the energy functional comprising both attracting and repelling forces. These critical measures are characterized by a cubic equation (spectral curve), and their components live on trajectories of a canonical quadratic differential on the Riemann surface of this equation. The structure of these trajectories and their deformations as function of α plays the crucial role.

This is a joint work with Guilherme L. Silva (University of Michigan, Ann Arbor).

Asymptotics of the recurrence coefficients of multiple orthogonal polynomials for Angelesco systems

05.03 **Maxim Yattselev**
(*Indiana University-Purdue University, Indianapolis, USA*)
Time: Monday 22.07., 11:30 - 12:00, Room HS 3

Abstract: In this talk I will describe asymptotics of the multiple orthogonal polynomials and their recurrence coefficients for an Anglesco system of two measures (measures are absolutely continuous with respect to the Lebesgue measure and have non-vanishing smooth densities) along all sequences of indices (n_1, n_2) for which n_1/n_2 has a limit (possibly infinite). Application to the recovery of the essential spectrum of a Jacobi operator on a 2-homogeneous rooted tree will be provided.

Joint work with A. I. Aptekarev and S. A. Denisov

On matrix Cauchy biorthogonal polynomials

05.04 **Sergio Medina Peralta**
(*Universidad Carlos III de Madrid, Spain*)
Time: Monday 22.07., 12:00 - 12:30, Room HS 3

Abstract: In this talk, we study the sequences of matrix Cauchy biorthogonal polynomials. We will focus on the algebraic aspects of the problem, finding a connection with a kind of mixed type Hermite-Padé approximation problem, and with its corresponding Riemann–Hilbert problem. From here we deduce some of the properties of these sequences of biorthogonal polynomials.

Some characterization problems related to d-orthogonal polynomial sets

05.05 **Hamza Chaggara**
(*Sousse University, Tunisia*)
Time: Monday 22.07., 15:30 - 16:00, Room HS 3

Abstract: The notion of d-orthogonal polynomials is a generalization of the notion of orthogonality in the sense that the polynomials $P_n,\ n = 0, 1, \ldots$ satisfy orthogonality conditions with respect to d forms. d-orthogonal polynomials are characterized by a higher-order recurrence relation of the form

$$P_{n+1}(x) = (x + \alpha_{n+1})P_n(x) + \sum_{k=0}^{d} \binom{n}{k} \beta_k^{(n+1)} P_{n-k}(x),\ \beta_d^{(n+1)} \neq 0\ n \geq 0.$$

We are interested, in this talk, with some characterization problems for d-orthogonal polynomial sequences when they satisfy additional properties. Indeed

- d-orthogonal polynomials of Sheffer type.
- Symmetric d-orthogonal polynomials of Sheffer type.
- Classical discrete d-orthogonal polynomials.

Matrix Laguerre biorthogonal polynomials via the Riemann–Hilbert Problem

05.06 **Ana Pilar Foulquié Moreno**
(*Universidade de Aveiro, Portugal*)
Time: Monday 22.07., 16:00 - 16:30, Room HS 3

Abstract: We use the Riemann–Hilbert problem, with jump supported on an appropriate curve in the complex plane, for characterizing the matrix biorthogonal polynomials. We apply this characterization for Matrix Laguerre weights and we derive the first and second order differential relations that the fundamental matrix, solution of the Riemann–Hilbert problem satisfies.

This is joint work with Amilcar Branquinho and Manuel Mañas.

On some multiple orthogonal polynomials of a discrete variable

05.07 **Jorge Arvesú Carballo**
(*Universidad Carlos III de Madrid, Spain*)
Time: Monday 22.07., 16:30 - 17:00, Room HS 3

Abstract: This presentation deals with algebraic and analytic properties of some discrete multiple orthogonal polynomials. First, introduce some special families of multiple orthogonal polynomials that are

q-analogous to discrete families given in [2]. Second, the raising and lowering operators, Rodrigues-type formula, and recurrence relations are discussed. Last, a connection with physical model involving weakly integrable systems [3] as well as with the weak asymptotics [1] for the studied families of multiple orthogonal polynomials will be shown.

This is a joint work with A. M. Ramírez-Aberasturis.

[1] A. I. Aptekarev, J. Arvesú, *Asymptotics for multiple Meixner polynomials*, J. Math. Anal. Appl. **411** (2014), 485–505.
[2] J. Arvesú, J. Coussement, W. Van Assche, *Some discrete multiple orthogonal polynomials*, J. Comput. Appl. Math. **153** (2003), 19–45.
[3] H. Miki, S. Tsujimoto, L. Vinet, A. Zhedanov, *An algebraic model for the multiple Meixner polynomials of the first kind*, J. Phys. A: Math. Theor. **45** (2012), 325205 (11 pp).

Multiple orthogonal polynomials associated with confluent hypergeometric functions

05.08

Hélder Lima
(*University of Kent, Canterbury, UK*)
Time: Monday 22.07., 17:00 - 17:30, Room HS 3

Abstract: In this talk we analyse a new family of multiple orthogonal polynomials of the hypergeometric type, whose orthogonality weights can be described using confluent hypergeometric functions (also known as Kummer or Tricomi functions) of the second kind, discuss some of their differential and recursive properties and exhibit an explicit formula for the polynomials. Particular cases are related with threefold symmetric multiple orthogonal polynomials.

This is ongoing work with Ana Loureiro.

Laguerre-Angelesco multiple orthogonal polynomials on an r-star

05.09

Marjolein Leurs
(*Katholieke Universitait Leuven, Belgium*)
Time: Tuesday 23.07., 10:30 - 11:00, Room HS 3

Abstract: The classical orthogonal polynomials consist of the Jacobi, Hermite and Laguerre polynomials. These can be generalized in a number of ways to multiple orthogonal polynomials which are orthogonal with respect to a system of r measures. In this talk we briefly state results of an extension of the Laguerre polynomials, the Laguerre-Angelesco multiple orthogonal polynomials. The Laguerre-Angelesco polynomials are orthogonal with respect to r measures which all have the same weight function $|x|^{\beta}e^{-x^r}$, each supported on a ray of an r-star. There are two types of multiple orthogonal polynomials. For each type we give explicit expressions for the polynomials, a differential equation and the asymptotic behavior of the zeros of the polynomials.

Multiple orthogonal polynomials living on a star: ratio asymptotics and zero limiting distributions

05.10

Ana Filipa Loureiro
(*University of Kent, Canterbury, UK*)
Time: Tuesday 23.07., 11:00 - 11:30, Room HS 3

Abstract: At the centre of this talk are polynomials satisfying higher order recurrence relations with all recurrence coefficients, except the last one, equal to zero. The polynomials at issue are orthogonal with respect to a vector of measures, are rotational invariant and all the zeros lie on a star in the complex plane. The discussion will focus on their ratio asymptotic behaviour as well as the zero limit distribution. The emphasis is on those polynomials whose recurrence coefficients are unbounded, but such that after scaling

become asymptotic periodic. The study is motivated and will be illustrated by examples of polynomials associated with confluent hypergeometric weight functions.

This is a joint work with Walter Van Assche.

Discrete multiple orthogonal polynomials on shifted lattices

05.11

Alexander Dyachenko
(*University College London, UK / Keldysh Institute of Applied Math., Moscow, Russia*)
Time: Tuesday 23.07., 11:30 - 12:00, Room HS 3

Abstract: There are many ways to define multiple orthogonal polynomials with respect to the classical continuous weights. The approach as in [1,2,3] preserves a kind of the Rodrigues formula, which is a very useful property. We focus on adapting this approach for the discrete case — bearing in mind the deep connection between the classical discrete and continuous orthogonality.

The talk is devoted to a new class of polynomials of multiple orthogonality with respect to the product of classical discrete weights on integer lattices with noninteger shifts. We obtain explicit representations in the form of the Rodrigues formulas. The case of two weights will be presented in more detail.

This is joint work with Vladimir Lysov.

[1] A. I. Aptekarev. Multiple orthogonal polynomials. *J. Comput. Appl. Math.* 99 (1998), no. 1–2, 423–447.
[2] A. I. Aptekarev, F. Marcellán, I. A. Rocha. Semiclassical multiple orthogonal polynomials and the properties of Jacobi-Bessel polynomials. *J. Approx. Theory* 90 (1997), no. 1, 117–146.
[3] W. Van Assche, E. Coussement. Some classical multiple orthogonal polynomials. *J. Comput. Appl. Math.* 127 (2001), no. 1–2, 317–347.

On the eigenvalues of Hermitian Brownian motion in critical situations

05.12

Thorsten Neuschel
(*Universität Bielefeld, Germany*)
Time: Tuesday 23.07., 12:00 - 12:30, Room HS 3

Abstract: We study the local behavior of the eigenvalues of Hermitian Brownian motion for large dimensions. These random eigenvalues form a determinantal point process for which in non-critical situations it is known that the local correlations show sine-kernel universality in the bulk of the spectrum. In this talk we focus on certain critical situations depending on the behavior of the initial configuration and show that the (multi-time) correlations exhibit Airy- or Pearcy-kernel universality.

The results presented are based on joint work with Tom Claeys and Martin Venker and they are part of a project which is still in progress.

Multiple Askey–Wilson polynomials and related multiple orthogonal polynomials

05.13

Walter Van Assche
(*Katholieke Universitait Leuven, Belgium*)
Time: Tuesday 23.07., 15:30 - 16:00, Room HS 3

Abstract: We first show how one can obtain Al-Salam–Chihara polynomials, continuous dual q-Hahn polynomials, and Askey–Wilson polynomials from the little q-Laguerre and the little q-Jacobi polynomials by using special transformations. This procedure is then extended to obtain multiple Askey–Wilson, multiple continuous dual q-Hahn, and multiple Al-Salam–Chihara polynomials from the multiple little q-Laguerre and the multiple little q-Jacobi polynomials.

This is joint work with Jean Paul Nuwacu (Université du Burundi)

Multilateral inversion of hypergeometric series

05.14 **Michael J. Schlosser**
(*University of Vienna, Austria*)
Time: Tuesday 23.07., 16:00 - 16:30, Room HS 3

Abstract: From Gustafson's A_r extension of Dougall's ${}_2H_2$ summation theorem we deduce a new explicit multilateral matrix inverse. As an application, we obtain a summation theorem for a specific A_r ${}_3H_3$ series.

A survey on biorthogonal polynomials and functions

05.15 **Clemente Cesarano**
(*Section of Mathematics – Uninettuno University, Roma, Italy*)
Time: Tuesday 23.07., 16:30 - 17:00, Room HS 3

Abstract: The theory of orthogonal polynomials is well established and detailed, covering a wide field of interesting results, as in particular for solving certain differential equations. On the other side the concepts and the related formalism of the theory of bi-orthogonal polynomials is less developed and much more limited. By starting from the orthogonality properties satisfied from the ordinary and generalized Hermite polynomials is possible to derive a further family (known in literature) of these kind of polynomials which are bi-orthogonal with their adjoint. This aspect allows us to introduce functions recognized as bi-orthogonal and investigate further generalizations of families of orthogonal polynomials. Furthermore, the use of the so called Chebyshev systems allows us to state relevant relations with bi-orthogonal polynomials with positive kernel.

Multiple orthogonal polynomials interpretation of some high order Toda systems

05.16 **Amilcar Branquinho**
(*Universidade de Coimbra, Portugal*)
Time: Tuesday 23.07., 17:00 - 17:30, Room HS 3

Abstract: Some discrete dynamical systems defined by a Lax pair are considered. The method of investigation is based on the analysis of the matrical moments for the main operator of the pair. The solutions of these systems are studied in terms of properties of this operator, giving, under some conditions, explicit expressions for the resolvent function.