On coherence relations between quasi-definite linear functionals and Sobolev orthogonal polynomials

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Abstract: In this talk we consider the non-coherence relation

$$\begin{aligned}&P_{n+1}^{[i]}(x)+a_n^{[1]}P_n^{[i]}(x)+a_n^{[2]}P_{n-1}^{[i]}(x)+b_n\left(Q_{n+1}(x)+c_nQ_n(x)\right)\\&=\left(1+b_n\right)R_{n+1}(x)+d_nR_n(x),\end{aligned} \tag{1}$$

where the sequences $\{P_n(x)\}_{n\geq 0}$, $\{Q_n(x)\}_{n\geq 0}$ and $\{R_n(x)\}_{n\geq 0}$ are orthogonal with respect to quasi-definite linear functionals u, v and w, respectively, with $P_k^{[i]}(x) := \frac{P_{k+i}^{(i)}(x)}{(k+1)_i}$, $i=0,1$, and $a_n^{[i]}b_nc_nd_n(1+b_n)\neq 0$, $n\geq 0$. Furthermore the linear functionals u and v are related through the rational relation $\rho u = v$ where $\deg\rho > 1$. We pointed out that (**??**) is linked to the concept of *symmetric* $(1,1)-$*coherent pairs* and, under certain conditions on the linear functionals, it can become a coherence relation. Under such conditions, we analyze an inverse problem associated with (**??**) as well as a direct problem where $i=1$, $a_n^{[2]}, c_n, d_n = 0$ and $b_n = 1$, for $n\geq 0$. Thereby we exhibit conditions under which the sequence $\{R_n(x)\}_{n\geq 0}$ is orthogonal with respect to a Borel positive measure μ supported on an infinite subset on the real line. The case when $\{Q_n(x)\}_{n\geq 0}$ and $\{P_n(x)\}_{n\geq 0}$ are the classical Chebyshev polynomials of the first and second kinds, respectively, is studied, as well as algebraic properties of the monic *Sobolev* polynomials, orthogonal with respect to the *Sobolev inner product*

$$\begin{aligned}\langle p,q\rangle_S &= \int_{-1}^{1} p(x)q(x)(1-x^2)^{-1/2}dx+\lambda_1\int_{-1}^{1}p'(x)q'(x)(1-x^2)^{1/2}dx\\&\quad+\lambda_2\int_{-1}^{1}p''(x)q''(x)d\mu(x),\end{aligned}$$

where $\lambda_1,\lambda_2>0$.