Rigorous numerical evaluation of D-finite functions in SageMath

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Time: Tuesday 23.07., 10:30 - 11:00, Room HS 5

Abstract: I will give a demo of the symbolic-numeric features available for working with D-finite functions in the Sage package `ore_algebra`.

Recall that a complex analytic function is called D-finite when it satisfies a linear ODE with polynomial coefficients. D-finite functions form a class analogous to that of hypergeometric functions, but more general. They come up in areas such as analytic combinatorics and mathematical physics, and lend themselves well to symbolic manipulation by computer algebra systems.

At the heart of the analytic features of `ore_algebra` is a rigorous implementation of numerical analytic continuation of D-finite functions. Numerical analytic continuation consists in computing numerical approximation of the transition matrices that maps initial values of an ODE somewhere on the complex plane to initial values elsewhere that define the same solution. The implementation is rigorous in the sense that it returns not just an approximation but an enclosure of the exact mathematical result.

Numerical analytic continuation is the basic brick for computing values of D-finite functions anywhere on their Riemann surfaces, rigorous polynomial approximations of D-finite functions on real or complex domains, monodromy matrices of differential operators, and other related objects. The code fully supports the important limit case where the (generalized) initial values are provided at regular singular points of the ODE, making it possible in particular to compute connection constants between regular singularities.