A new symmetric representation of the differential equation for the Laguerre-Sobolev polynomials

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Abstract: In the enduring fruitful research on orthogonal polynomials in weighted Sobolev spaces, the Laguerre-Sobolev polynomials are playing a prominent role. They are orthogonal with respect to a Sobolev-type inner product associated with the classical Laguerre measure on the positive half-line and two point masses $M, N > 0$ at the origin involving functions and their first derivatives. A particularly useful feature of the Laguerre-Sobolev polynomials is their property to arise as the eigenfunctions of a spectral differential operator which, for any Laguerre parameter $\alpha \in \mathbb{N}_0$, is of finite order $4\alpha + 10$. The main purpose of this talk is to establish a new symmetric form of this differential operator which consists of a number of elementary components depending on α, M, N. In particular, the new representation enables us to deduce the symmetry of the operator in the Sobolev space. This readily recovers the orthogonality of the polynomial eigenfunctions. The present results have strongly been motivated and guided by our recent observations that the higher-order differential equations for the so-called Bochner-Krall orthogonal polynomials possess an elementary symmetric form which differs considerably from the classical Lagrange symmetric form. Among them is the differential equation of order $2\alpha + 2\beta + 6$ for the generalized Jacobi polynomials which will serve us as a pattern to illustrate how the main features of the equation carry over to the Laguerre-Sobolev case. We close with some new promising results on the Jacobi-Sobolev equations.