On the irrationality and the measure of irrationality of $\log(1+1/m)\log(1-1/m)$

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Abstract: We consider the Diophantine approximants for the product of the two logarithms $\gamma_m := \log\left(1+\frac{1}{m}\right)\log\left(1-\frac{1}{m}\right)$ for an integer m. We prove that for all $m \geq 33$ the number γ_m is irrational. This is an improvement of the previous result by M. Hata [1]. We also find new upper estimates of the measure of irrationality of γ_m.

Our approach is based on the Hermite–Padé approximants for the vector of functions (f_1, f_2, f_3), where

$$f_1(z) := \log\left(1+\frac{1}{z}\right), \quad f_2(z) := \log\left(1-\frac{1}{z}\right), \quad f_3 := f_1 f_2.$$

This vector is an example of the Generalized Nikishin system of Markov functions on graphs [2]. The common denominator of the approximants satisfies certain multiple orthogonality relations. The key ingredient of our proof is an explicit formula for the common denominator. By means of this formula we obtain the asymptotics of the sequence of the approximants and also some remarkable arithmetic properties of them.

[1] M. Hata. The irrationality of $\log(1+1/q)\log(1-1/q)$. *Trans. Amer. Math. Soc.* **350**:6 (1998), 2311–2327.
[2] A. I. Aptekarev, V. G. Lysov. Systems of Markov functions generated by graphs and the asymptotics of their Hermite-Padé approximants. *Mat. Sb.* **201**:2 (2010), 183–234.