Exceptional extensions of some $q = -1$ classical orthogonal polynomials

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Abstract: In recent years, significant progress on exceptional orthogonal polynomial systems has been made by researchers from mathematical and physical aspects. It was proved that every system of exceptional orthogonal polynomials can be obtained from a classical orthogonal polynomial system through a sequence of Darboux transformations. Note that the classical orthogonal polynomials mentioned in this context are the Hermite, Laguerre and Jacobi polynomials, sometimes also be referred to as the "very" classical orthogonal polynomials. In general, polynomials in the Askey-Wilson scheme all can be called classical. The term classical means that apart from a three-term recurrence relation, these polynomials satisfy also an eigenvalue equation. Recently, several new families of polynomial systems which appear by taking a nontrivial limit $q = -1$ on orthogonal polynomials from the Askey-Wilson scheme have been identified classical. They satisfy eigenvalue problems with differential/difference operators of Dunkl type. Specifically, these polynomials are the Bannai-Ito polynomials, the big -1-Jacobi and the little -1-Jacobi polynomials. Unlike the previous cases, the associated Dunkl-type operators are of first-order which cannot be factorized into two first-order as it was performed in an ordinary Darboux transformation. Therefore, we apply a generalized Darboux transformation by making use of a pair of intertwining relations satisfied by the Dunkl-type operators. In this way we derive the exceptional extensions of these $q = -1$ polynomials. An interesting fact of these exceptional orthogonal polynomial systems is that in several cases the corresponding degree sequences are consist of even numbers only, for example, $\{0, 2, 2, 4, 4, \dots\}$. We further study their ladder operators and the associated algebraic relations to address this fact. This is joint work with Satoshi Tsujimoto, Luc Vinet, and Alexei Zhedanov.