Hypergeometric form of fundamental theorem of calculus

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Time: Monday 22.07., 16:00 - 16:30, Room AM

Abstract: We introduce a natural method of computing antiderivatives of a large class of functions which stems from the observation that the series expansion of an antiderivative differs from the series expansion of the corresponding integrand by just two Pochhammer symbols. All antiderivatives are thus, in a sense, "hypergeometric". And hypergeometric functions are therefore the most natural functions to integrate.

In this talk we would like to make two points: First, the method presented is *easy*. So much so that it can be taught in undergraduate university level. And second: It may be used to prove some of the more challenging examples.

In particular, we show that

$$\int_0^\infty \sqrt[8]{\frac{x^2+8x+8-4(2+x)\sqrt{1+x}}{x^{11}}}\mathrm{d}x = \frac{4\Gamma^2\left(\frac{1}{4}\right)}{3\sqrt{2-\sqrt{2}}\sqrt{\pi}},$$

$$G = \Re\left({}_3F_2\left(\begin{matrix} 1 & 1 & 1 \\ & 2 & 2 \end{matrix}; \mathrm{i}\right)\right) = \Im\left(\left[\epsilon^2\right]{}_2F_1\left(\begin{matrix} \epsilon & \epsilon \\ & 1 \end{matrix}; \mathrm{i}\right)\right) = \frac{1}{8}\left(\psi'\left(\frac{1}{4}\right) - \pi^2\right),$$

where G is the Catalan's constant and

$$\int_0^1 x \ln\frac{1}{1+x^2} K(\mathrm{i}x)\mathrm{d}x = \frac{1}{4\sqrt{2\pi}}\left((2-\ln 2)\Gamma^2\left(\frac{1}{4}\right) + 4(\ln 2 - 4)\Gamma^2\left(\frac{3}{4}\right)\right),$$

where K is the complete elliptic integral of the first kind. All of this using a single technique.