Biorthogonal rational functions involving two parameters and the Christoffel type transformation

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Abstract: In this work, a general T-fraction based on a polynomial map is considered. Two generalized linear matrix pencils of the form $\mathcal{G} - z\mathcal{H}$, where $\mathcal{G}$ and $\mathcal{H}$ are tridiagonal matrices, associated to this polynomial map are considered and the orthogonality of the related Laurent polynomials are discussed. These matrix pencils are useful in constructing two sequences of biorthogonal rational functions, $\{p_n^L(z)\}_{n=0}^{\infty}$ and $\{p_n^R(z)\}_{n=0}^{\infty}$, associated with the parameters a_n and b_n respectively, that form the components of the left and right eigenvectors of the matrix pencil. The procedure for constructing these two families is different from the one given in [3]. These two different sequences of orthogonal rational functions lead to the recurrence relations given by

$$\mathcal{P}_{n+1}(z) = \rho_n(z-\nu_n)\mathcal{P}_n(z) + \tau_n(z-a_n)(z-b_n)\mathcal{P}_{n-1}(z), n \geq 1,$$

with initial conditions $\mathcal{P}_0(z) = 1$ and $\mathcal{P}_1(z) = \rho_0(z-\nu_0)$ that are defined on the unit circle as well in the real line. These are known as R_{II} type recurrence relations and were studied by Ismail and Masson [2] and Zhedanov [4] independently. A particular case is considered that provides a Christoffel type transformation of the generalized eigenvalue problem with a reformulation different from the existing literature. Specific illustrations are provided to support the given results.

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[4] A. Zhedanov, Biorthogonal rational functions and the generalized eigenvalue problem, J. Approx. Theory **101** (1999), no. 2, 303–329.