Transition between characters of classical groups, decomposition of Gelfand-Tsetlin patterns and last passage percolation

(joint work with Nikos Zygouras)

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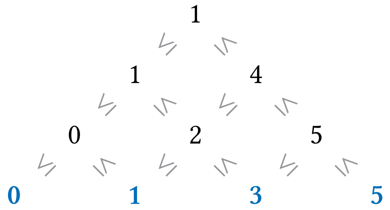
Summary

Schur polynomials of type A

Gelfand-Tsetlin pattern
$z = \{z_{i,j}\}_{1 \le j \le i \le n}$ of height n:

- $z_{i,j} \in \mathbb{Z}_{\ge 0}$
- interlacing conditions
- shape $\boldsymbol{\lambda} = z_n$



Schur polynomial of type A

$$s_{\lambda}(x_1, \ldots, x_n) := \sum_{\mathrm{sh}(z)=\lambda} \prod_{i=1}^{n} x_i^{|z_i| - |z_{i-1}|}$$

where $|z_i| := z_{i,1} + z_{i,2} + \ldots$.

Invariance: under permutations of x_i's.

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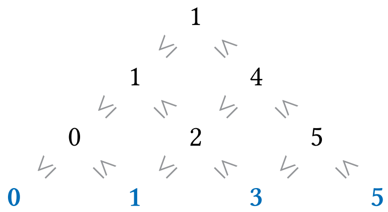

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Schur polynomials of type C

Symplectic pattern
$z = \{z_{i,j}\}_{1 \le i \le 2n, 1 \le j \le \lceil i/2 \rceil}$
of height $2n$:

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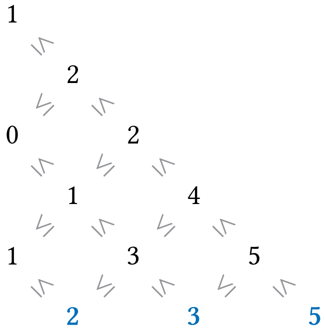

Schur polynomial of type C (characters of $\mathrm{Sp}_{2n}(\mathbb{C})$)

$$\mathrm{sp}_{\lambda}(x_1, \ldots, x_n) := \sum_{\mathrm{sh}(z)=\lambda} \prod_{i=1}^{n} x_i^{|z_{2i}|-|z_{2i-1}|} (x_i^{-1})^{|z_{2i-1}|-|z_{2i-2}|}$$

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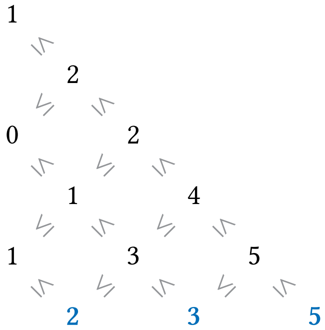

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Schur polynomials of type B

Split orthogonal pattern
$z = \{z_{i,j}\}_{1\le i\le 2n, 1\le j\le \lceil i/2\rceil}$
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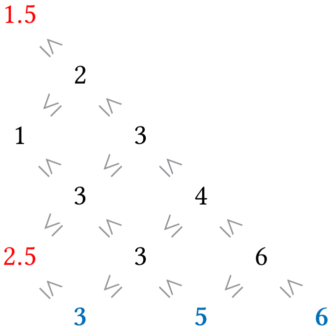

Schur polynomial of type B (characters of $SO_{2n+1}(\mathbb{C})$) [Proctor, 1994]

$$\mathrm{so}_\lambda(x_1,\dots,x_n) := \sum_{\mathrm{sh}(z)=\lambda} \prod_{i=1}^{n} x_i^{|z_{2i}|-|z_{2i-1}|} \left(x_i^{-1}\right)^{|z_{2i-1}|-|z_{2i-2}|}$$

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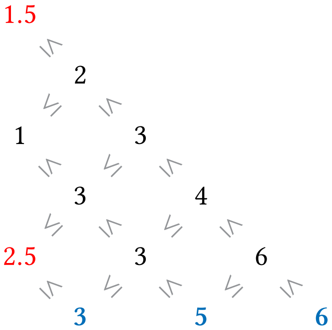

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CB-interpolating Schur polynomials

CB-interpolating Schur polynomial (with parameter β)

$$s^{\mathrm{CB}}_{\lambda}(x;\beta) := \sum_{\mathrm{sh}(z)=\lambda} \beta^{\#\{i:\, z_{i,2i-1}\in\frac{1}{2}+\mathbb{Z}_{\geq 0}\}} \prod_{i=1}^{n} x_i^{|z_{2i}|-|z_{2i-1}|} \left(x_i^{-1}\right)^{|z_{2i-1}|-|z_{2i-2}|}$$

Interpolation between characters of type C and B:

$$s^{\mathrm{CB}}_{\lambda}(x;\beta) = \begin{cases} \mathrm{sp}_{\lambda}(x) & \text{if } \beta = 0 \\ \mathrm{so}_{\lambda}(x) & \text{if } \beta = 1 \end{cases}$$

Theorem [B.-Zygouras, 2019]

$$s^{\mathrm{CB}}_{\lambda}(x;\beta) = \sum_{\substack{\lambda/\mu \\ \text{vert. strip}}} \beta^{|\lambda/\mu|}\, \mathrm{sp}_{\mu}(x)$$

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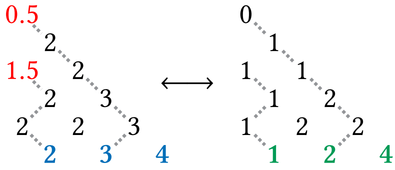

Determinantal formula

Corollary [B.-Zygouras, 2019]

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$$= \frac{\det_{1\le i,j\le n}\left(x_j^{\lambda_i+n-i+1} - x_j^{-(\lambda_i+n-i+1)} + \beta\left[x_j^{\lambda_i+n-i} - x_j^{-(\lambda_i+n-i)}\right]\right)}{\det_{1\le i,j\le n}\left(x_j^{n-i+1} - x_j^{-(n-i+1)}\right)}$$

- invariance under permutation of x_i's and inversion $x_i \mapsto x_i^{-1}$
- specialisation of Koornwinder polynomials:

$$s_{\boldsymbol{\lambda}}^{\mathrm{CB}}(\boldsymbol{x};\beta) = K_{\boldsymbol{\lambda}}(\boldsymbol{x};0,0;\beta,0,0,0)$$

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Identities of CB-interpolating polynomials

Theorem [B.-Zygouras, 2019]

For $u \in \frac{1}{2}\mathbb{Z}_{\geq 0}$, the following quantities are equal:

$$B := \sum_{\mu \subseteq (2u)^{2n}} \beta^{\mathrm{oddrows}\,\mu} \cdot s_{\mu}(x_1, \dots, x_{2n})$$

$$C := \left[\prod_{i=1}^{2n} x_i\right]^{u} s^{\mathrm{CB}}_{u^{2n}}(x_1, \dots, x_{2n}; \beta)$$

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- $B = C$ $\begin{cases} \beta = 0\text{: [Stembridge, 1990]} \\ \beta = 1\text{: [Macdonald, 1979]} \\ \text{general } \beta\text{: consequence of [Krattenthaler, 1998]} \end{cases}$
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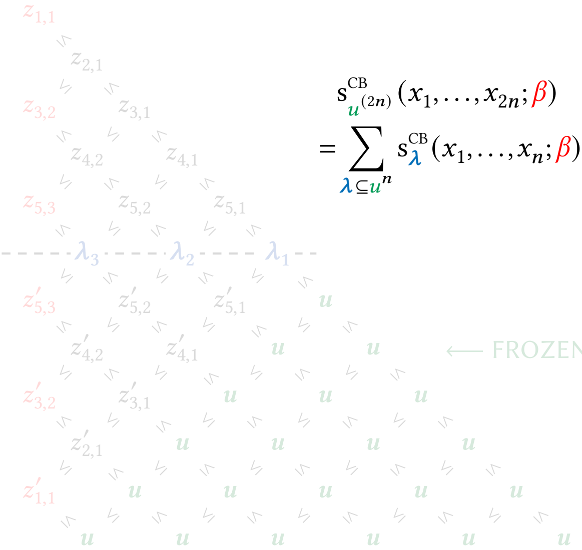
Pictorial proof of C=D



$$s^{CB}_{u^{(2n)}}(x_1,\dots,x_{2n};\beta) = \sum_{\lambda\subseteq u^n} s^{CB}_{\lambda}(x_1,\dots,x_n;\beta)\cdot s^{CB}_{\lambda}(x_{n+1},\dots,x_{2n};\beta)$$

$\longleftarrow$ FROZEN PART OF u's

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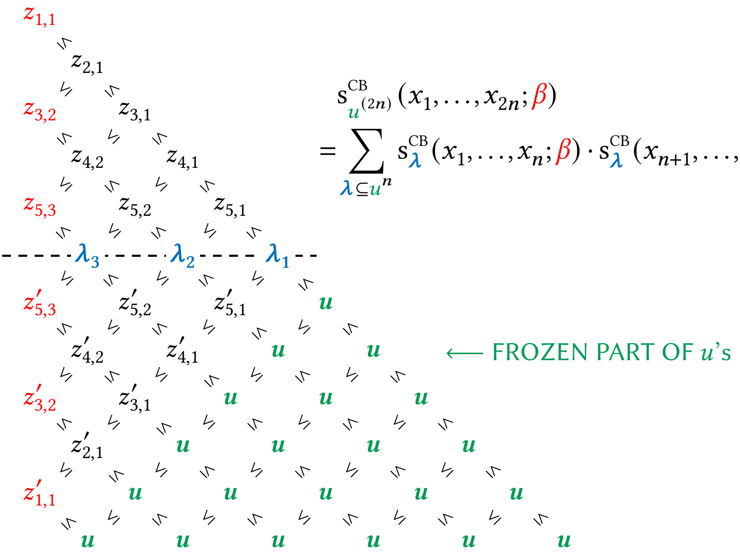

$$
\begin{aligned}
&s^{\mathrm{CB}}_{u^{(2n)}}(x_1,\ldots,x_{2n};\beta) \\
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\end{aligned}
$$

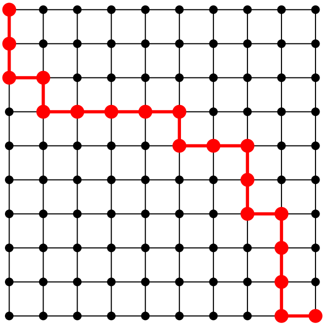
Point-to-point last passage percolation

- Random weights:

$$\{W_{i,j}\}_{1\leq i,j\leq N}$$

- Directed point-to-point path from $(1,1)$ to (N,N):

$$\pi \in \Pi_{N,N}$$



Last passage percolation (LPP)

$$L(N,N) := \max_{\pi\in\Pi_{N,N}} \sum_{(i,j)\in\pi} W_{i,j}$$

- Integrable model with geometric weights: [Johansson, 2000]
- Model from the *KPZ universality class*: fluctuations of order $\sqrt[3]{N}$ and asymptotic distribution from random matrix theory

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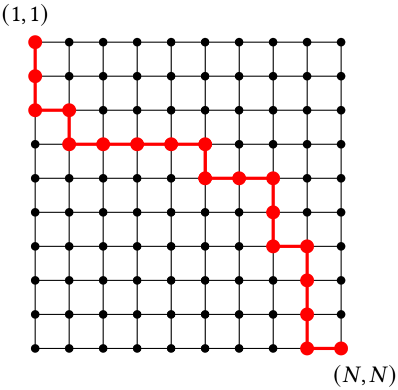

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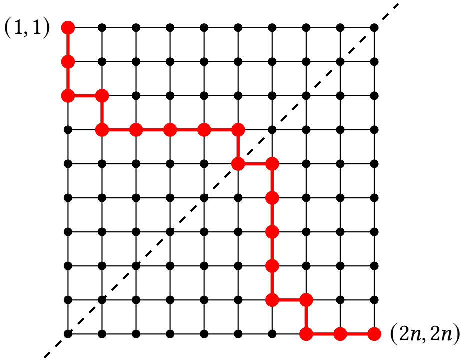

$L_\beta^{\boxslash}(2n, 2n)$

$L_\beta^{\triangledown}(2n, 2n) = \frac{1}{2} L_\beta^{\boxslash}(2n, 2n)$

$$\mathbb{P}(W_{i,j} = k) = \begin{cases} (1 - x_{2n-i+1}x_j)(x_{2n-i+1}x_j)^k & \text{if } i+j < 2n+1 \\ \dfrac{1 - x_j^2}{1 + \beta x_j} \beta^{k \bmod 2} x_j^k & \text{if } i+j = 2n+1 \end{cases}$$

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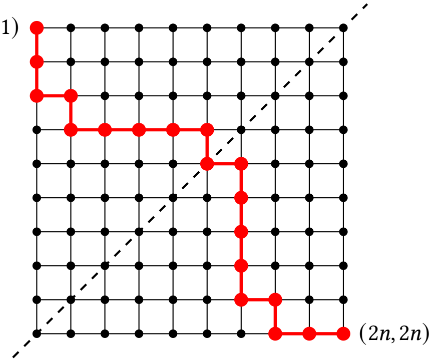

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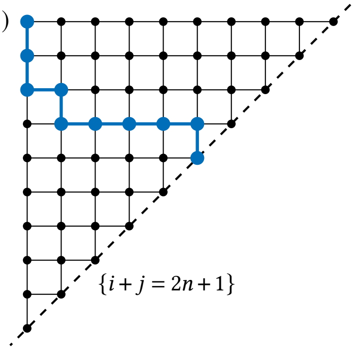

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Point-to-line LPP distribution

Theorem

For $u \in \frac{1}{2}\mathbb{Z}_{\geq 0}$, the following quantities are equal:

$$A := c_{\beta}^{\boxslash} \cdot \mathbb{P}\left(L_{\beta}^{\boxslash}(2n,2n) \leq 2u\right)$$

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- $A = D$: [B.-Zygouras, 2019] via RSK on triangular arrays

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A Pfaffian-determinant duality

$$\lim_{n\to\infty} \mathbb{P}\left(L_{\beta}^{\boxslash}(2n,2n) \le c_1 n + c_2 s \sqrt[3]{n}\right) = F_1(s)$$

Point-to-line LPP	Symmetric point-to-point LPP
↓ RSK on triangular arrays	↓ RSK on square arrays
CB-interpolating polynomials	Schur polynomials
↓ Cauchy-Binet identity	↓ de Bruijn identity
Determinantal formula	Pfaffian formula
↓ Sylvester's identity	↓ Sylvester's identity
Fredholm determinant	Fredholm Pfaffian
↓ steepest descent	↓ steepest descent
GOE Tracy-Widom law	GOE Tracy-Widom law

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References

This talk was based on:

- E. Bisi and N. Zygouras. Transition between characters of classical groups, decomposition of Gelfand-Tsetlin patterns, and last passage percolation. *Adv. Math.*, 404(B), 108453 (2022).

Other references:

- J. Baik and E. M. Rains. Algebraic aspects of increasing subsequences. *Duke Math. J.*, 109(1):1–65 (2001).
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