Computing positroid cells in the Grassmannian of lines, their boundaries and their intersections

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Positroids are families of matroids introduced by Postnikov in the study of non-negative Grassmannians. In particular, positroids enumerate a CW decomposition of the totally non-negative Grassmannian. Furthermore, Postnikov has identified several families of combinatorial objects in bijections with positroids. However, to date, there is no practical algorithm to determine whether a given set represents (the dependent sets of) a positroid, and if so what is its relation with other positroids. We will provide yet another characterization of positroids for $\mathrm{Gr}_{\geq 0}(2,n)$, the Grassmannians of lines, in terms of certain graphs. We will use this characterization to answer to the following questions: given a collection $\mathcal{D}$ of 2-subsets of $[n] := \{1, ..., n\}$, is $\mathcal{D}$ the dependent sets of a matroid? Is there a $2 \times n$ matrix whose columns i and j are (linearly) dependent if and only if $\{i, j\} \in \mathcal{D}$? Does there exist such a matrix with non-negative 2-minors? How can we determine which positroids lie in the boundary of a given positroid cell? This also leads to a combinatorial description of the intersection of positroid cells, that is easily computable. Our techniques rely on determining different ways to enlarge a given collection of subsets of $\{1, \ldots, n\}$ to represent the dependent sets of a positroid, that is the dependencies among the columns of a matrix with non-negative maximal minors. Furthermore, we provide an algorithm to compute all the maximal matroids and all the maximal positroids contained in a set.

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