Tutte's invariants

01.03 **Mireille Bousquet-Mélou**
(*Université de Bordeaux*)
Time: Monday 04.07., 11:00 – 11:50

Abstract: The enumeration of many classes of combinatorial objects (lattice paths, maps, permutations...), according to their size, can be performed by recording additional statistics, now often called "catalytic". In this talk, we will focus on examples for which a recursive decomposition of the objects yields a functional equation for their generating function that involves "divided differences" of the form $(F(x) - F(0))/x$, where x is one of the catalytic variables. More precisely, we focus on examples with two catalytic variables x and y, and two divided differences.

Typical recent examples come from the enumeration of plane latice walks confined to a cone. But the historical example is a functional equation for planar triangulations equipped with a proper colouring, written by William Tutte in 1973. It took him about 10 years to solve this equation, and prove that its solution satisfies an ordinary differential equation. In this solution, he defined and used a crucial notion of **invariants**.

I will give a modern view of invariants, and show recent applications to walks confined to a cone.