Panel Discussion: Robotics and other challenges for computer supported geometrical reasoning

Tomas Recio
http://www.recio.tk
Universidad Antonio de Nebrija

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https://www3.risc.jku.at/conferences/adg2021/?content=general

Challenges

- "Citius, Altius, Fortius!" (eg. P. Quaresma competition!)

- Linkage automated analysis and design

- Cooperation with education stakeholders: Is there Euclidean geometry in education? Competencies, PBL, IBL, STEAM…

- AR: visually impaired people, math trails, art analysis

From M. Husty's talk

Interesting open Problems

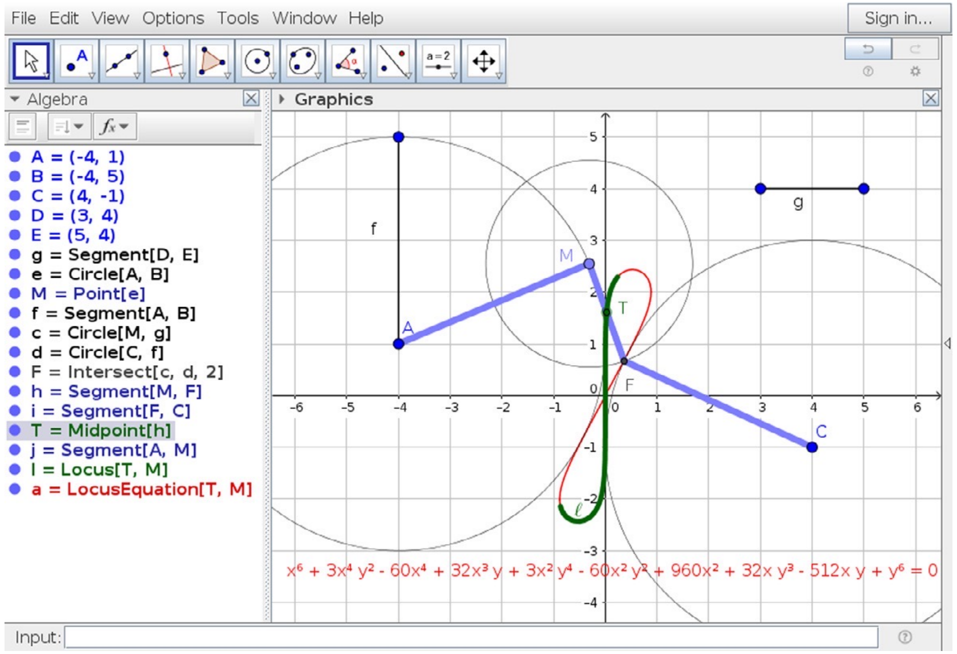
- Can dynamics be formulated within the algebraic framework?
- Complete solution of planar and spatial wire robots?
- Multiplicity of solutions.
- Global path planning of mobile robots.
- Overconstrained 6-R classification - underconstrained extreme n-R linkages.

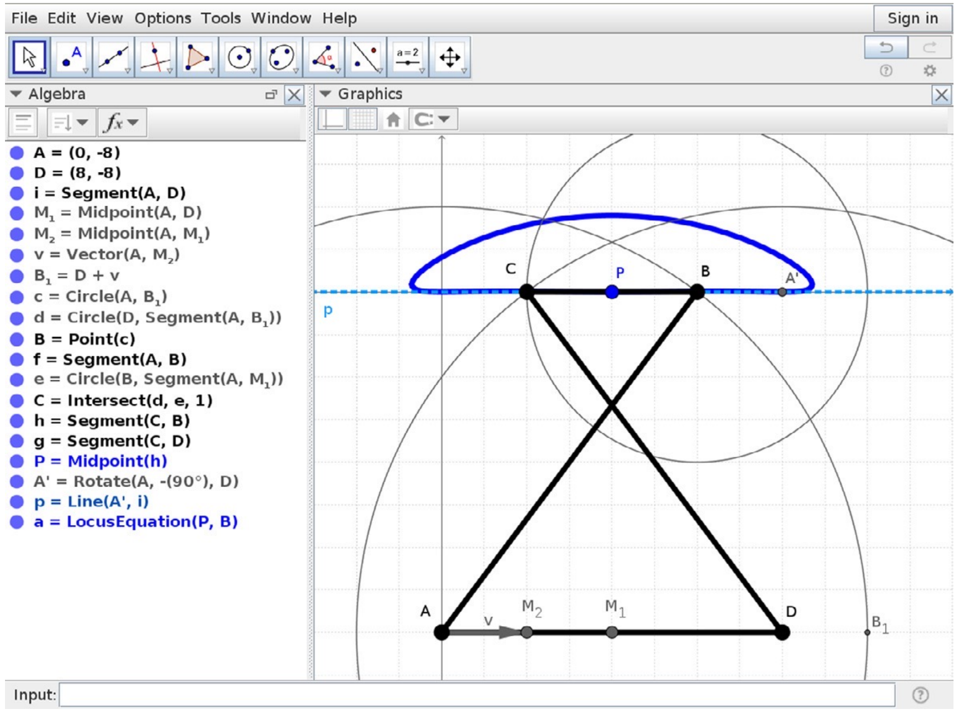
Linkages:

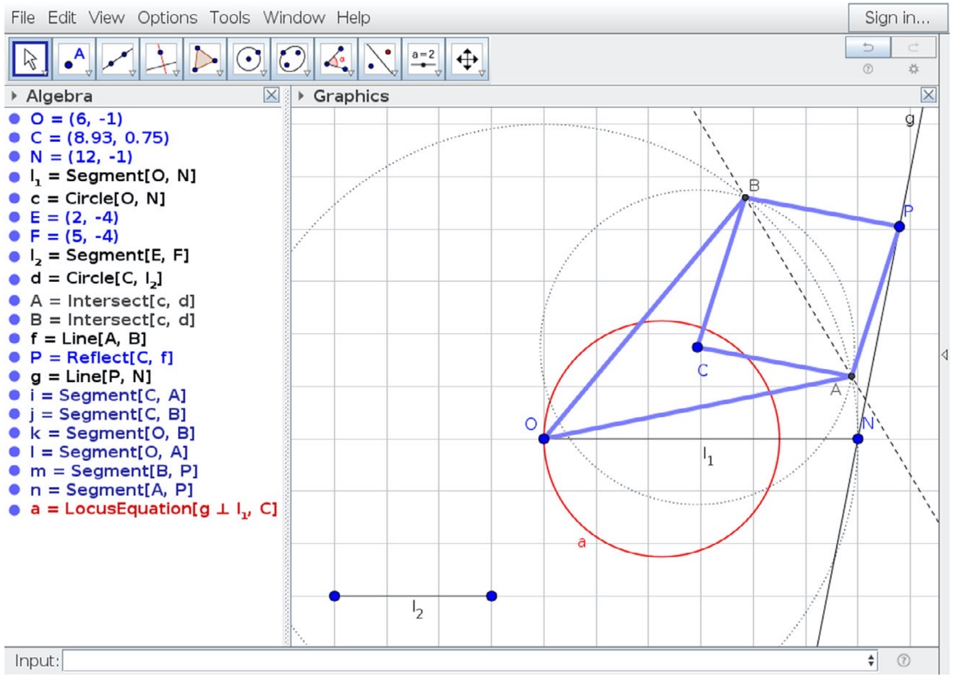
Automatic derivation of locus equation

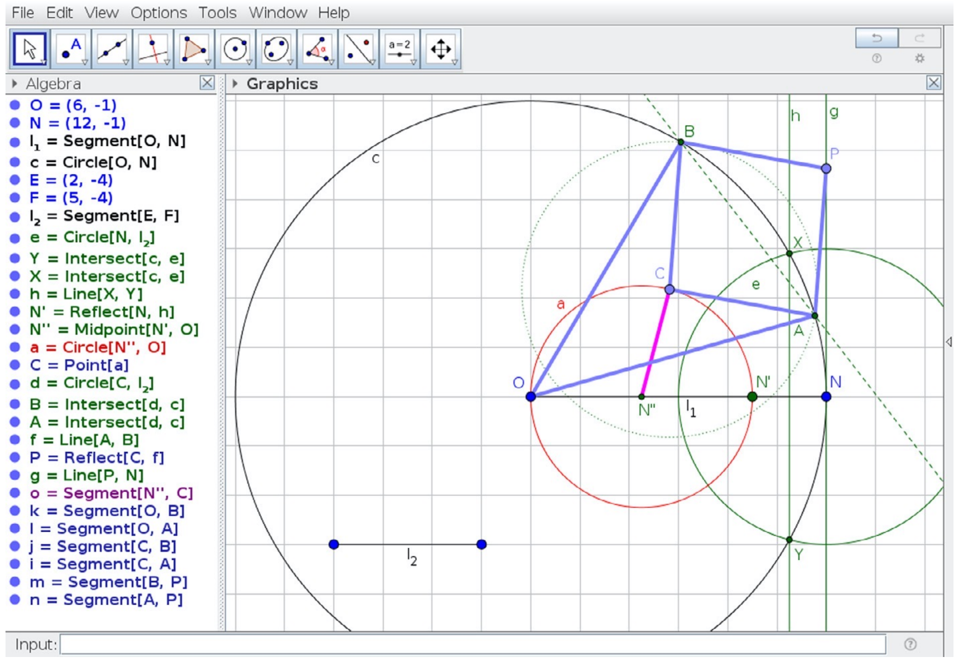
Verification of correctness of the expected performance

Automatic redesign for achieving a desired movement


File Edit View Options Tools Window Help
Sign in...
Algebra
Graphics
A = (-4, 1)
B = (-4, 5)
C = (4, -1)
D = (3, 4)
E = (5, 4)
g = Segment[D, E]
e = Circle[A, B]
M = Point[e]
f = Segment[A, B]
c = Circle[M, g]
d = Circle[C, f]
F = Intersect[c, d, 2]
h = Segment[M, F]
i = Segment[F, C]
T = Midpoint[h]
j = Segment[A, M]
l = Locus[T, M]
a = LocusEquation[T, M]
x⁶ + 3x⁴ y² - 60x⁴ + 32x³ y + 3x² y⁴ - 60x² y² + 960x² + 32x y³ - 512x y + y⁶ = 0
Input:


File Edit View Options Tools Window Help
Sign in
Algebra
Graphics
A = (0, -8)
D = (8, -8)
i = Segment(A, D)
M1 = Midpoint(A, D)
M2 = Midpoint(A, M1)
v = Vector(A, M2)
B1 = D + v
c = Circle(A, B1)
d = Circle(D, Segment(A, B1))
B = Point(c)
f = Segment(A, B)
e = Circle(B, Segment(A, M1))
C = Intersect(d, e, 1)
h = Segment(C, B)
g = Segment(C, D)
P = Midpoint(h)
A' = Rotate(A, -(90°), D)
p = Line(A', i)
a = LocusEquation(P, B)
Input:





Zoltán Kovács, Tomás Recio, M. Pilar Vélez: *Reasoning about linkages with dynamic geometry*, Journal of Symbolic Computation 97 (2020) 16–30

Cooperation with education stakeholders

The key is to make a start, beginning with exploratory studies of the potential of these new tools at both the secondary and post-secondary levels.

https://en.wikipedia.org/wiki/Gila_Hanna

Gila Hanna and Xiaoheng (Kitty) Yan: **Opening a discussion on teaching proof with automated theorem provers,** For the Learning of Mathematics, Nov. 2021.

- ***GeoGebra's automated proving tools***

GeoGebra …has gained in popularity over the last twenty years and is now widely used… GeoGebra has recently added an Automated Reasoning Tool (ART) to help students conjecture that a given property holds for a specific geometric object and then to find a proof that their conjecture is true. If that is not the case and the property does not hold, ART can also help students make the necessary changes to the original conjecture (Hohenwarter, Kovács, & Recio, 2019, p. 216).

Since the developers of GeoGebra added reasoning tools to their software, they have published a large number of papers in scholarly journals describing the potential of those tools for secondary-school learning…These additions appear to benefit students at both the undergraduate and the secondary level.

It is perhaps too early for empirical studies of classroom experience using the enhancements to GeoGebra… While it is reasonable to expect proof technology to foster students' proving abilities, and there is certainly supporting anecdotal evidence, its potential advantages have not yet been systematically assessed.

Proof assistants that meet the requirements of these stakeholders

(*the curriculum decision makers (who specify the standard of mathematical validation at a given grade), the teachers (who orchestrate learning and decide what counts as a proof in relation to a standard), and the learners (who are simultaneously constructing an understanding of proof and of the related content*) Balacheff & Boy de la Tour

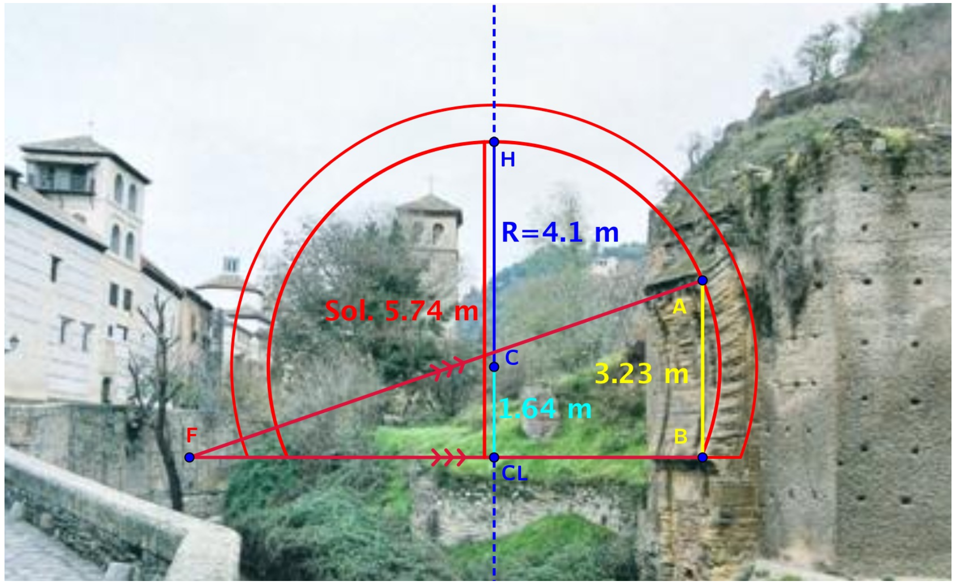
will never be developed in the absence of initiative on the part of mathematics educators and a demonstrated demand fuelled by increased use. Secondly, success also requires new and effective teaching strategies. These two efforts stand in a reciprocal relationship, so that the full benefit of proof assistants will be seen only over time as new teaching strategies effect the demand for new tool features and vice versa. The responsibility for both efforts rests squarely on the shoulders of educators

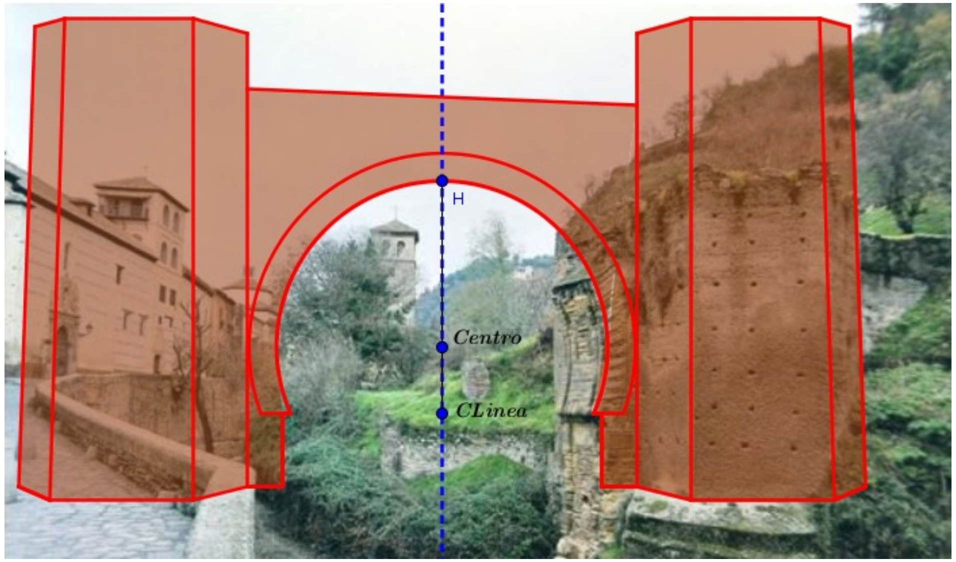
- Hohenwarter, M.; Kovacs, Z.; Recio, T. : "*Using GeoGebra Automated Reasoning Tools to explore geometric statements and conjectures*". In Hanna, G., de Villiers, M., Reid, D. (Eds.), Proof Technology in Mathematics Research and Teaching, Series: Mathematics Education in the Digital Era, Vol. 14, 2019, p. 215-236. Springer Cham. https://doi.org/10.1007/978-3-030-28483-1_10

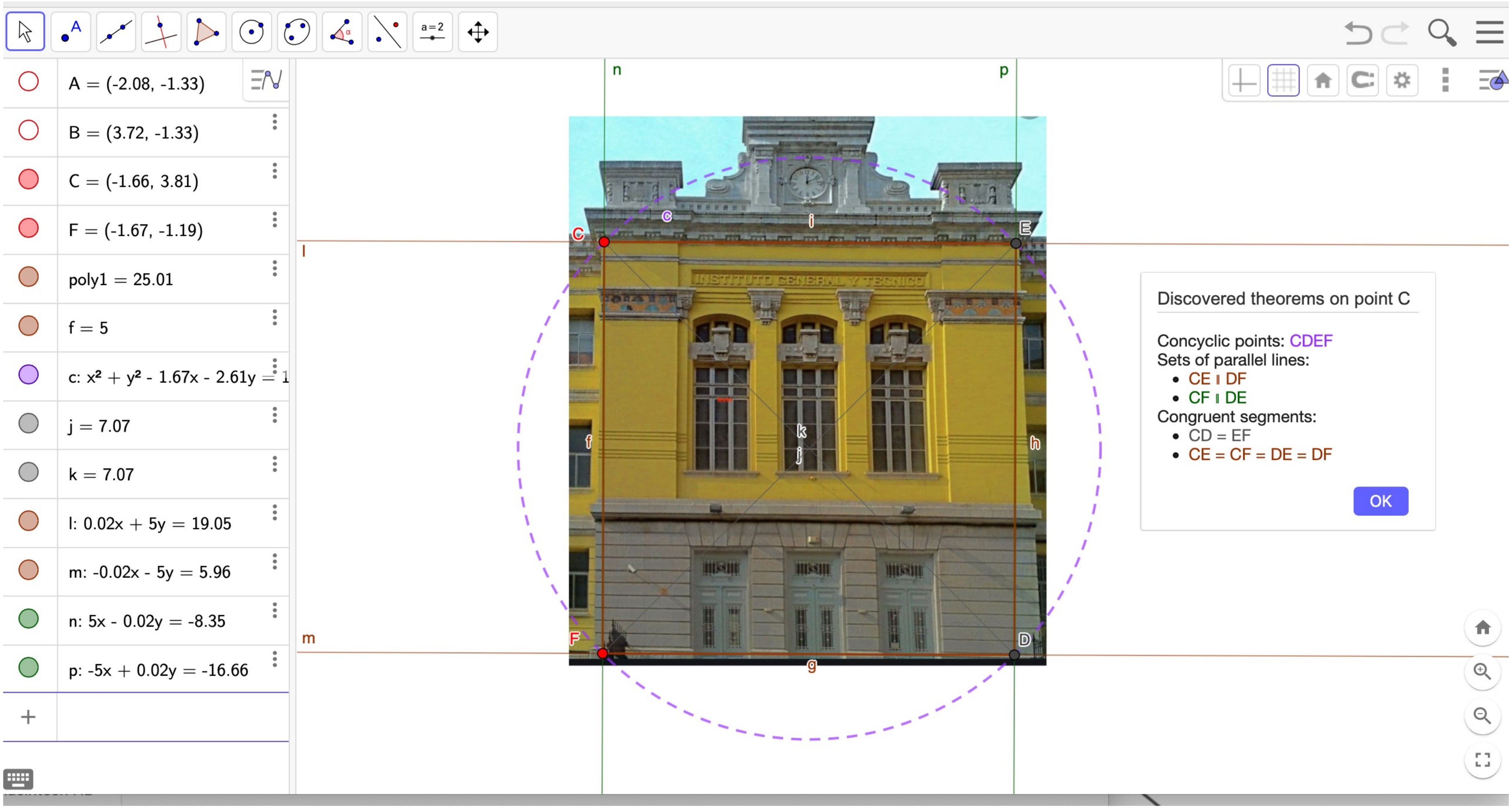
- Kovács, Z.; Recio, T.; Richard, P.R.; Van Vaerenbergh, S.; Vélez, M.P.: "*Towards an Ecosystem for Computer-Supported Geometric Reasoning*". International Journal of Mathematical Education in Science and Technology. Nov. 2, 2020 (on-line). https://doi.org/10.1080/0020739X.2020.1837400

- Kovács, Z.; Recio, T.; Velez, M.P. : "*Automated Reasoning Tools with GeoGebra: What are they? What are they good for?*" In: Mathematics Education in the Age of Artificial Intelligence; Richard, P.R., Vélez, M.P., Van Vaerenbergh, S., Eds.; Series: Mathematics Education in the Digital Era; Springer Nature Switzerland AG, 2022

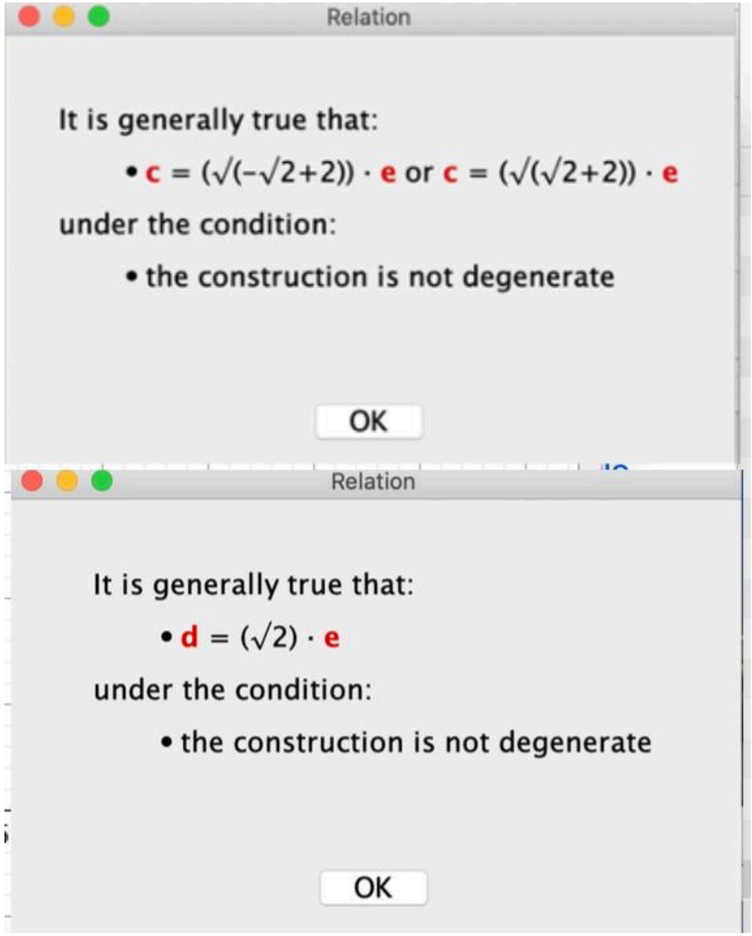
AR: visually impaired people, math trails, art analysis

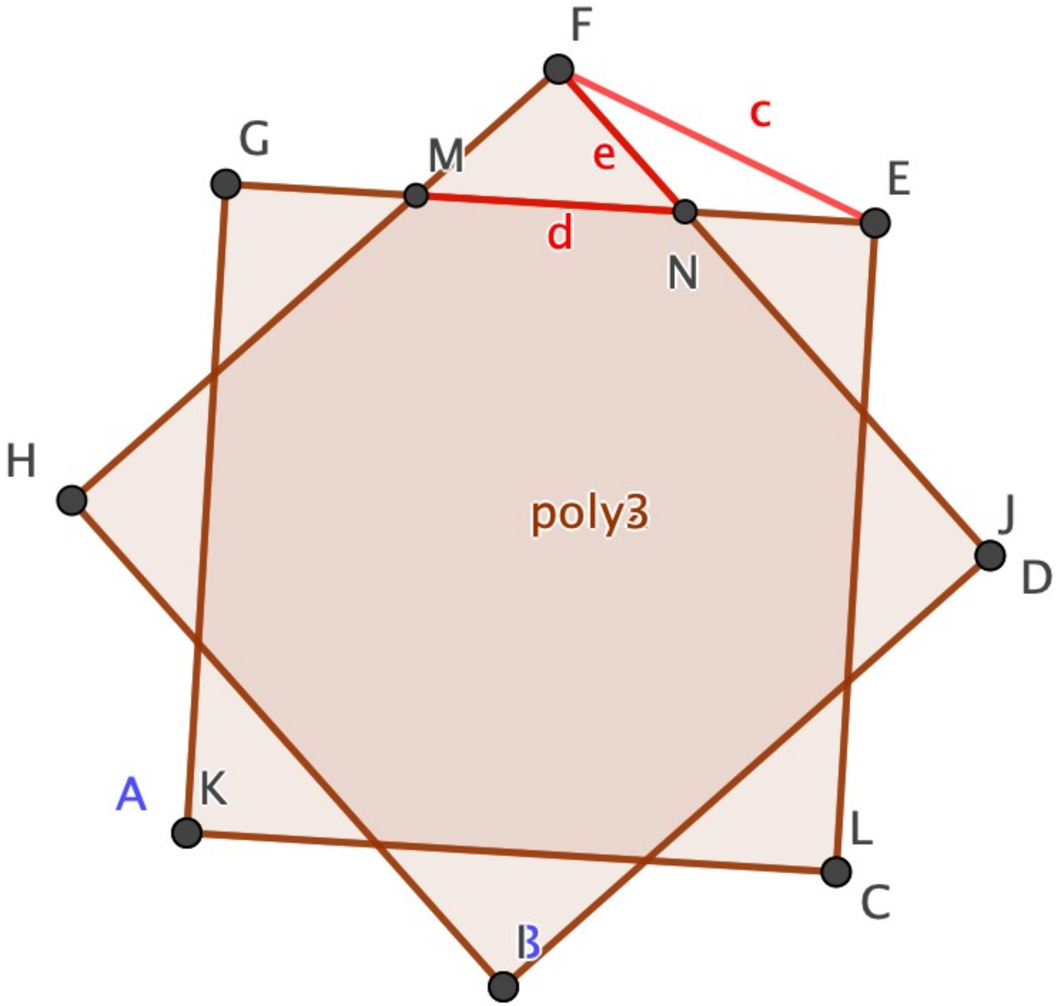

H
R=4.1 m
Sol. 5.74 m
C
A
3.23 m
1.64 m
B
F
CL


H
Centro
CLinea


A = (-2.08, -1.33)
B = (3.72, -1.33)
C = (-1.66, 3.81)
F = (-1.67, -1.19)
poly1 = 25.01
f = 5
c: x² + y² - 1.67x - 2.61y = 1
j = 7.07
k = 7.07
l: 0.02x + 5y = 19.05
m: -0.02x - 5y = 5.96
n: 5x - 0.02y = -8.35
p: -5x + 0.02y = -16.66
INSTITUTO GENERAL Y TECNICO
Discovered theorems on point C
Concyclic points: CDEF
Sets of parallel lines:
CE ∥ DF
CF ∥ DE
Congruent segments:
CD = EF
CE = CF = DE = DF
OK


c
e
d


Relation
It is generally true that:
• c = (√(−√2+2)) · e or c = (√(√2+2)) · e
under the condition:
• the construction is not degenerate
OK
Relation
It is generally true that:
• d = (√2) · e
under the condition:
• the construction is not degenerate
OK


F
c
G
M
e
E
d
N
H
poly3
J
D
A
K
L
C
B

- Botana F.; Kovács Z., Martinez-Sevilla, A. and Recio T.:"Automatically Augmented Reality with GeoGebra", In: Theodosia Prodromou (Ed.), Augmented Reality in Educational Settings, Brill-Sense. Nov. 2019. https://doi.org/10.1163/9789004408845

- Botana F.; Kovács Z. and Recio T.: "Automatically Augmented Reality for Outdoor Mathematics". In: *Research on Outdoor STEM Education in the digiTal Age. Proceedings of the ROSETA Online Conference in June 2020.* Matthias Ludwig, Simone Jablonski, Amélia Caldeira and Ana Moura (Editors). WTM – Verlag für wissenschaftliche Texte und Medien, Münster 2020. Conference Proceedings in Mathematics Education (6), pages 71-78. https://doi.org/10.37626/GA9783959871440.0

THANKS

http://www.recio.tk