Verification techniques for cryptographic protocols

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Context : cryptographic protocols

- Widely used : web (SSH, SSL, ...), pay-per-view, electronic purse, mobile phone, ...
- Should ensure : confidentiality, authenticity, integrity, anonymity, ...

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- Widely used : web (SSH, SSL, ...), pay-per-view, electronic purse, mobile phone, ...
- Should ensure : confidentiality, authenticity, integrity, anonymity, ...
- Presence of an attacker
  - may read every message sent on the net,
  - may intercept and send new messages.

Example : Credit Card Payment Protocol

- The waiter introduces the credit card.
- The waiter enters the amount m of the transaction on the terminal.
- The terminal authenticates the card.
- The customer enters his secret code.
  If the amount m is greater than 100 euros
  (and in only 20% of the cases)
  - The terminal asks the bank for authentication of the card.
  - The bank provides authentication.

More details

4 actors : Bank, Customer, Card and Terminal.

Bank owns

- a signing key K_B^{-1}, secret,
- a verification key K_B, public,
- a secret symmetric key for each credit card K_{CB}, secret.

Card owns

- Data : last name, first name, card's number, expiration date,
- Signature's Value $VS = \{hash(\text{Data})\}_{K_B^{-1}}$,
- secret key K_{CB}.

Terminal owns the verification key K_B for bank's signatures.

Credit card payment Protocol (in short)

The terminal reads the card :

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The terminal calls the bank :

$$\begin{array}{llll} 5. & T & \rightarrow & B : auth? \\ 6. & B & \rightarrow & T : N_b \\ 7. & T & \rightarrow & Ca : N_b \\ 8. & Ca & \rightarrow & T : \{N_b\}_{K_{CB}} \\ 9. & T & \rightarrow & B : \{N_b\}_{K_{CB}} \\ 10. & B & \rightarrow & T : ok \end{array}$$

Some flaws

The security was initially ensured by :

- the cards were very difficult to reproduce,
- the protocol and the keys were secret.

But

- cryptographic flaw : 320 bits keys can be broken (1988),
- logical flaw : no link between the secret code and the authentication of the card,
- fake cards can be build.

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→ "YesCard" build by Serge Humpich (1998).

How does the "YesCard" work ?

Logical flaw

$$
\begin{array}{llcll}
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2. & T & \rightarrow & Ca & : secret\ code? \\
3. & Cu & \rightarrow & Ca & : 1234 \\
4. & Ca & \rightarrow & T & : ok
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$$

How does the “YesCard” work ?

Logical flaw

$$
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1. & Ca & \rightarrow T & : \mathrm{Data}, \{hash(\mathrm{Data})\}_{K_B^{-1}} \\
2. & T & \rightarrow Ca & : secret\ code? \\
3. & Cu & \rightarrow Ca' & : 2345 \\
4. & Ca' & \rightarrow T & : ok
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Remark : there is always somebody to debit.
→ creation of a fake card (Serge Humpich).

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$\rightarrow$ creation of a fake card (Serge Humpich).

$$\begin{array}{llll}
1. & Ca' & \rightarrow T & : \text{XXX}, \{hash(\text{XXX})\}_{K_B^{-1}} \\
2. & T & \rightarrow Cu & : secret\ code? \\
3. & Cu & \rightarrow Ca' & : 0000 \\
4. & Ca' & \rightarrow T & : ok
\end{array}$$

Outline of the talk

Motivation : Cryptography does not suffice to ensure security !

Example : Commutative encryption (RSA)

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Since $\left\{\{\text{pin} : 3443\}_{k_{alice}}\right\}_{k_{bob}} = \left\{\{\text{pin} : 3443\}_{k_{bob}}\right\}_{k_{alice}}$

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$$\xrightarrow{\{\text{pin} : 3443\}_{k_{\text{alice}}}}$$

$$\xleftarrow{\left\{\{\text{pin} : 3443\}_{k_{\text{alice}}}\right\}_{k_{\text{intruder}}}}$$

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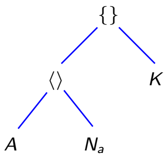
Messages

Messages are abstracted by terms.

Agents : $a, b, \ldots$ Nonces : $n_1, n_2, \ldots$ Keys : $k_1, k_2, \ldots$
Cyphertext : $\{m\}_k$ Concatenation : $\langle m_1, m_2 \rangle$

Example : The message $\{A, N_a\}_K$ is represented by :



Intruder abilities

Composition rules

$$\frac{T \vdash u \quad T \vdash v}{T \vdash \langle u\,,v\rangle} \qquad \frac{T \vdash u \quad T \vdash v}{T \vdash \mathrm{enc}(u,v)} \qquad \frac{T \vdash u \quad T \vdash v}{T \vdash \mathrm{enca}(u,v)}$$

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Decomposition rules

$$\frac{}{T \vdash u}\ u \in T \qquad \frac{T \vdash \langle u\ ,\, v \rangle}{T \vdash u} \qquad \frac{T \vdash \langle u\ ,\, v \rangle}{T \vdash v}$$

$$\frac{T \vdash \mathrm{enc}(u, v) \quad T \vdash v}{T \vdash u} \qquad \frac{T \vdash \mathrm{enca}(u, \mathrm{pub}(v)) \quad T \vdash \mathrm{priv}(v)}{T \vdash u}$$

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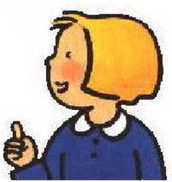
Deducibility relation

A term u is deducible from a set of terms T, denoted by $T \vdash u$, if there exists a prooftree witnessing this fact.

A simple protocol

$$\xrightarrow{\langle \mathsf{Bob}, \mathsf{k} \rangle}$$

$$\xleftarrow{\langle \mathsf{Alice}, \mathsf{enc}(\mathsf{s}, \mathsf{k}) \rangle}$$



A simple protocol

$\xrightarrow{\langle \mathsf{Bob}, \mathsf{k} \rangle}$

$\xleftarrow{\langle \mathsf{Alice}, \mathsf{enc}(\mathsf{s}, \mathsf{k}) \rangle}$

Question ?

Can the attacker learn the secret s ?

A simple protocol

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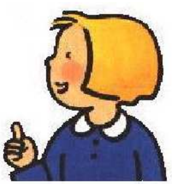

Answer : Of course, Yes !

$$\dfrac{\dfrac{\langle \mathsf{Alice}, \mathsf{enc}(\mathsf{s}, \mathsf{k})\rangle}{\mathsf{enc}(\mathsf{s}, \mathsf{k})} \qquad \dfrac{\langle \mathsf{Bob}, \mathsf{k}\rangle}{\mathsf{k}}}{\mathsf{s}}$$

Decision of the intruder problem

Given A set of messages S and a message m

Question Can the intruder learn m from S that is $S \vdash m$?

This problem is decidable in polynomial time

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Lemma (Locality)

If there is a proof of $S \vdash m$ then there is a proof that only uses the subterms of S and m.

Protocol description

Protocol :

$$\begin{array}{rcl} A \rightarrow B & : & \{\text{pin}\}_{k_a} \\ B \rightarrow A & : & \{\{\text{pin}\}_{k_a}\}_{k_b} \\ A \rightarrow B & : & \{\text{pin}\}_{k_b} \end{array}$$

A protocol is a finite set of roles :

- role $\Pi(1)$ corresponding to the 1[st] participant played by a talking to b :

$$\begin{array}{rcl} \text{init} & \xrightarrow{k_a} & \text{enc}(\text{pin}, k_a) \\ \text{enc}(x, k_a) & \rightarrow & x. \end{array}$$

Protocol description

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- role $\Pi(2)$ corresponding to the 2^{nd} participant played by b with a :

$$\begin{array}{rcl} x & \xrightarrow{k_b} & \mathrm{enc}(x, k_b) \\ \mathrm{enc}(y, k_b) & \rightarrow & \mathrm{stop}. \end{array}$$

Secrecy via constraint solving

Constraint systems are used to specify secrecy preservation under a particular, finite scenario.

Scenario

$$
\begin{array}{ccc}
\mathrm{rcv}(u_1) & \xrightarrow{N_1} & \mathrm{snd}(v_1) \\
\mathrm{rcv}(u_2) & \xrightarrow{N_2} & \mathrm{snd}(v_2) \\
 & \ldots & \\
\mathrm{rcv}(u_n) & \xrightarrow{N_n} & \mathrm{snd}(v_n)
\end{array}
$$

Constraint System

$$
\mathcal{C} = \begin{cases}
T_0 \Vdash u_1 \\
T_0, v_1 \Vdash u_2 \\
\quad \ldots \\
T_0, v_1, .., v_n \Vdash s
\end{cases}
$$

Remark : Constraint Systems may be used more generally for trace-based properties, e.g. authentication.

Secrecy via constraint solving

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Scenario

$$\begin{array}{c} \mathsf{rcv}(u_1) \xrightarrow{N_1} \mathsf{snd}(v_1) \\ \mathsf{rcv}(u_2) \xrightarrow{N_2} \mathsf{snd}(v_2) \\ \dots \\ \mathsf{rcv}(u_n) \xrightarrow{N_n} \mathsf{snd}(v_n) \end{array}$$

Constraint System

$$\mathcal{C} = \begin{cases} T_0 \Vdash u_1 \\ T_0, v_1 \Vdash u_2 \\ \quad \dots \\ T_0, v_1, .., v_n \Vdash s \end{cases}$$

Solution of a constraint system

A substitution σ such that

for every $T \Vdash u \in \mathcal{C}$, $u\sigma$ is deducible from $T\sigma$, that is $u\sigma \vdash T\sigma$.

How to solve constraint system ?

Given $\mathcal{C} = \begin{cases} T_0 \Vdash u_1 \\ T_0, v_1 \Vdash u_2 \\ \quad \ldots \\ T_0, v_1, .., v_n \Vdash u_{n+1} \end{cases}$

Question Is there a solution σ of $\mathcal{C}$?

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Advertisement :
Lecture of Hubert Comon-Lundh at ISR 2008 next week

An easy case : "solved constraint systems"

Given $\mathcal{C} = \begin{cases} T_0 \Vdash x_1 \\ T_0, v_1 \Vdash x_2 \\ \quad \ldots \\ T_0, v_1, .., v_n \Vdash x_{n+1} \end{cases}$

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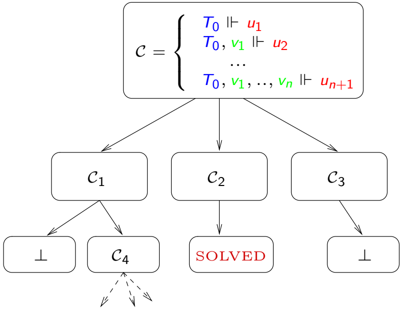
Question Is there a solution σ of $\mathcal{C}$?

Of course yes !
Consider e.g. $\sigma(x_1) = \cdots = \sigma(x_{n+1}) = t \in T_0$.

Decision procedure [Millen / Comon-Lundh]

Goal : Transformation of the constraints in order to obtain a solved constraint system.



$\mathcal{C}$ has a solution iff $\mathcal{C} \rightsquigarrow \mathcal{C}'$ with $\mathcal{C}'$ in solved form.

Intruder step

The intruder can built messages

$$R_5 : \quad \mathcal{C} \wedge T \Vdash f(u, v) \quad \rightsquigarrow \quad \mathcal{C} \wedge T \Vdash u \wedge T \Vdash v$$
$$\text{for } f \in \{\langle\rangle, \mathrm{enc}, \mathrm{enca}\}$$

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Example :

$$a, k \Vdash \mathrm{enc}(\langle x, y\rangle, k) \quad \rightsquigarrow \quad \begin{array}{l} a, k \Vdash k \\ a, k \Vdash x \\ a, k \Vdash y \end{array}$$

Eliminating redundancies

$$k \Vdash x$$
$$\mathsf{enc}(s, x) \Vdash s$$

The constraint $\mathsf{enc}(s, x) \Vdash s$ will be satisfied as soon as $k \Vdash x$ is satisfied.

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The constraint $\mathrm{enc}(s, x) \Vdash s$ will be satisfied as soon as $k \Vdash x$ is satisfied.

$$R_1 : \mathcal{C} \wedge T \Vdash u \rightsquigarrow \mathcal{C} \quad \text{if } T \cup \{x \mid T' \Vdash x \in \mathcal{C}, T' \subsetneq T\} \vdash u$$

Unsolvable constraints

$$R_4 : \mathcal{C} \wedge T \Vdash u \rightsquigarrow \bot \qquad \text{if } \mathrm{var}(T, u) = \emptyset \text{ and } T \not\vdash u$$

Example :

$$\begin{array}{lcl} \ldots & & \\ a, \mathrm{enc}(s, k) \Vdash s & \rightsquigarrow & \bot \\ \ldots & & \end{array}$$

Guessing equalities

1. Example : $k, \mathrm{enc}(\mathrm{enc}(x, k'), k) \Vdash \mathrm{enc}(a, k')$

$$R_2 : \mathcal{C} \ \wedge \ T \Vdash u \rightsquigarrow_\sigma \mathcal{C}\sigma \ \wedge \ T\sigma \Vdash u\sigma \qquad u' \in st(T)$$
$$\text{if } \sigma = \mathrm{mgu}(u, u'), \ u, u' \notin \mathcal{X}, \ u \neq u'$$

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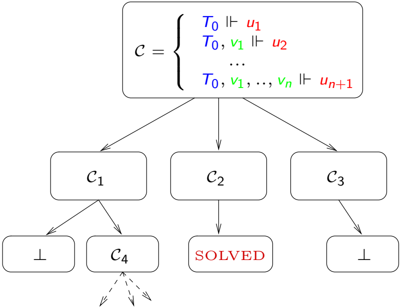
2. Example : $\mathrm{enc}(s, \langle a, x \rangle), \mathrm{enc}(\langle y, b \rangle, k), k \Vdash s$

$$R_3 : \mathcal{C} \wedge T \Vdash v \rightsquigarrow_\sigma \mathcal{C}\sigma \wedge T\sigma \Vdash v\sigma \qquad u, u' \in st(T)$$
$$\text{if } \sigma = \mathrm{mgu}(u, u'),\ u, u' \notin \mathcal{X},\ u \neq u'$$

NP-procedure for solving constraint systems



Theorem

- *$\mathcal{C}$ has a solution iff $\mathcal{C} \rightsquigarrow \mathcal{C}'$ with $\mathcal{C}'$ in solved form.*
- *$\rightsquigarrow$ is terminating in polynomial time.*

What formal methods allow to do ?

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- For a bounded number of sessions, secrecy is co-NP-complete [RusinowitchTuruani CSFW01]
  → numerous tools for detecting attacks (Casper, Avispa platform...)
- For an unbounded number of sessions
  - for one-copy protocols, secrecy is DEXPTIME-complete [CortierComon RTA03] [SeildVerma LPAR04]
  - for message-length bounded protocols, secrecy is DEXPTIME-complete [Durgin et al FMSP99] [Chevalier et al CSL03]

  → some tools for proving security (ProVerif, EVA Platform)

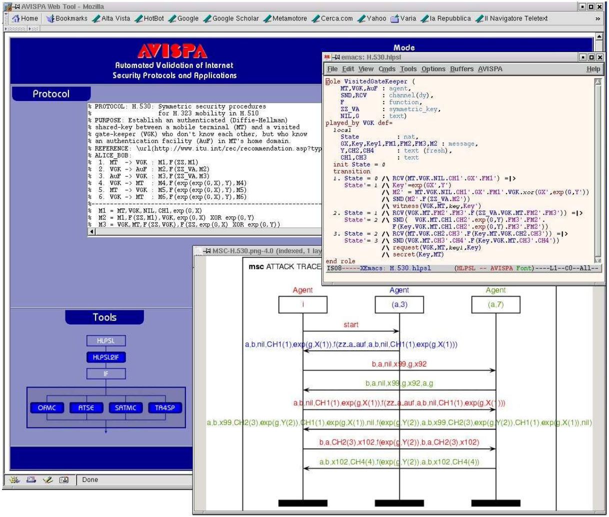
Tools

Many tools for a bounded number of sessions (search for attacks) : Casper, Avispa platform, ...

Some tools for an unbounded number of sessions (security proof) : ProVerif, EVA platform

- new attacks have been discovered (e.g. the man-in-the-middle attack on the Needham-Schroeder protocol)
- hundreds protocols analyzed in few minutes or few seconds for most of them
- real-world applications (IETF, ...)

Example of tool : Avispa Platform



Collaborators

- Cassis project, Loria
- DIST, Italy
- ETHZ, Swiss
- Siemens, Germany

www.avispa-project.org

Outline of the talk

Motivation

Back to our running example :

$$\begin{array}{rcl} A \rightarrow B & : & \{\mathrm{pin}\}_{k_a} \\ B \rightarrow A & : & \{\{\mathrm{pin}\}_{k_a}\}_{k_b} \\ A \rightarrow B & : & \{\mathrm{pin}\}_{k_b} \end{array}$$

We need the equation for the commutativity of encryption

$$\{\{z\}_x\}_y = \{\{z\}_y\}_x$$

Some other examples

Encryption-Decryption theory

$$\mathrm{dec}(\mathrm{enc}(x,y),y) = x \quad \pi_1(\langle x,y\rangle) = x \quad \pi_2(\langle x,y\rangle) = y$$

EXclusive Or

$$\begin{array}{rclrcl} x \oplus (y \oplus z) & = & z & x \oplus y & = & y \oplus x \\ x \oplus x & = & 0 & x \oplus 0 & = & x \end{array}$$

Diffie-Hellmann

$$\exp(\exp(z,x),y) = \exp(\exp(z,y),x)$$

E-voting protocols

First phase :

$$V \rightarrow A : \quad \mathrm{sign}(\mathit{blind}(\mathrm{vote}, r), V)$$
$$A \rightarrow V : \quad \mathrm{sign}(\mathit{blind}(\mathrm{vote}, r), A)$$

Voting phase :

$$V \rightarrow C : \quad \mathrm{sign}(\mathrm{vote}, A)$$

...

Equational theory for blind signatures

[Kremer Ryan 05]

$$\begin{array}{rcl}
\mathrm{checksign}(\mathrm{sign}(x,y),\mathrm{pk}(y)) & = & x \\
\mathrm{unblind}(\mathrm{blind}(x,y),y) & = & x \\
\mathrm{unblind}(\mathrm{sign}(\mathrm{blind}(x,y),z),y) & = & \mathrm{sign}(x,z)
\end{array}$$

Deduction

$$\frac{}{T \vdash_E M}\ M \in T \qquad \frac{T \vdash_E M_1 \quad \cdots \quad T \vdash_E M_k}{T \vdash_E f(M_1, \ldots, M_k)}\ f \in \Sigma$$

$$\frac{T \vdash M}{T \vdash M'}\ M =_E M'$$

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$$\frac{T \vdash M}{T \vdash M'}\ M =_E M'$$

Example : $E := \mathrm{dec}(\mathrm{enc}(x, y), y) = x$ and $T = \{\mathrm{enc}(secret, k),\ k\}$.

$$\dfrac{\dfrac{\dfrac{}{T \vdash \mathrm{enc}(secret, k)} \qquad \dfrac{}{T \vdash k}}{T \vdash \mathrm{dec}(\mathrm{enc}(secret, k), k)}\ f \in \Sigma}{T \vdash secret}\ \mathrm{dec}(\mathrm{enc}(x, y), y) = x$$

Rewriting systems

For analyzing equational theories, we (try to) associate to E a finite convergent rewriting system $\mathcal{R}$ such that :

$$u =_E v \quad \text{iff} \quad u\downarrow = v\downarrow$$

Definition (Characterization of the deduction relation)

Let $t_1, \ldots t_n$ and u be terms in normal form.

$$\{t_1, \ldots t_n\} \vdash u \quad \text{iff} \quad \exists C \text{ s.t. } C[t_1, \ldots, t_n] \rightarrow^* u$$

(Also called Cap Intruder problem [Narendran et al])

Some results with equational theories

	Security problem	
	Bounded number of sessions	**Unbounded number of sessions**
Commutative encryption	*co-NP-complete* [CKRT04]	**Ping-pong protocols :** *co-NP-complete* [Turuani04]
Exclusive Or	*Decidable* [CS03,CKRT03]	**One copy - No nonces :** *Decidable* [CLC03] **Two-way automata - No nonces :** *Decidable* [Verma03]
Abelian Groups	*Decidable* [Shmatikov04]	**Two-way automata - No nonces :** *Decidable* [Verma03]
Prefix encryption	*co-NP-complete* [CKRT03]	
Abelian Groups and Modular Exponentiation	**General case :** *Decidable* [Shmatikov04] **Restricted protocols :** *co-NP-complete* [CKRT03]	**AC properties of the Modular Exponentiation No nonces :** *Semi-Decision Procedure* [GLRV04]

Outline of the talk

Specificity of cryptographic models

- Messages are bitstrings
- Real encryption algorithm
- Real signature algorithm
- General and powerful adversary

$\longrightarrow$ very little abstract model

Encryption nowadays

→ Based on algorithmically hard problems.

RSA Function $n = pq$, p et q primes.

e : public exponent

- $x \mapsto x^e \mod n$ easy (cubic)
- $y = x^e \mapsto x \mod n$ difficult
  $x = y^d$ où $d = e^{-1} \mod \phi(n)$

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Diffie-Hellman Problem

- Given $A = g^a$ and $B = g^b$,
- Compute $\mathrm{DH}(A, B) = g^{ab}$

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Diffie-Hellman Problem

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- Compute $\mathrm{DH}(A, B) = g^{ab}$

→ Based on hardness of integer factorization.

Setting for cryptographic protocols

Protocol :

- Message exchange program
- using cryptographic primitives

Adversary $\mathcal{A}$: any probabilistic polynomial Turing machine, *i.e.* any probabilistic polynomial program.

- polynomial : captures what is feasible
- probabilistic : the adversary may try to guess some information

Definition of secrecy preservation

→ Several notions of secrecy :

One-Wayness : The probability for an adversary $\mathcal{A}$ to compute the secret s against a protocol $\mathcal{P}$ is negligible (smaller than any inverse of polynomial).

$$\forall p \text{ polynomial } \quad \exists \eta_0 \ \forall \eta \geq \eta_0 \quad \mathrm{Pr}^{\eta}_{m,r}[\mathcal{A}(\mathcal{P}_K) = s] \leq \frac{1}{p(\eta)}$$

η : security parameter = key length

Not strong enough !

- The adversary may be able to compute half of the secret message.
- There is no guarantee in case that some partial information on the secret is known.

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→ Introduction of a notion of indistinguishability.

Indistinguishability

The secrecy of s is defined through the following game :

- Two values n_0 and n_1 are randomly generated instead of s ;
- The adversary interacts with the protocol where s is replaced by n_b, $b \in \{0, 1\}$;
- We give the pair (n_0, n_1) to the adversary ;
- The adversary gives b',

The data s is secret if $\Pr[b = b'] - \frac{1}{2}$ is a negligible function.

Formal and Cryptographic approaches

	Formal approach	Cryptographic approach
Messages	terms	bitstrings
Encryption	idealized	algorithm
Adversary	idealized	any polynomial algorithm
Secrecy property	reachability-based property	indistinguishability
Guarantees	unclear	strong
Protocol	complex, several sessions	simple, one session

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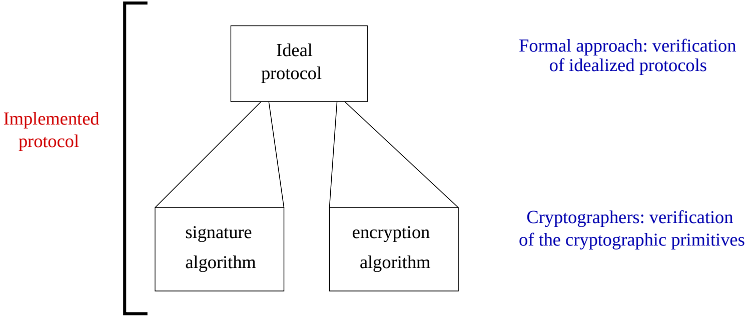
Guarantees	unclear	strong
Protocol	complex, several sessions	simple, one session
Proof	automatic	by hand, tedious and error-prone

Link between the two approaches ?

Composition of the two approaches

Automatic cryptographically sound proofs



→ Currently implemented in the Avispa platform.

Example : correspondence of secrecy properties

Theorem

Symbolic secrecy implies cryptographic indistinguishability.

- For protocols with only public key encryption, signatures and nonces
- Provided the public key encryption and the signature algorithms verify strong existing cryptographic properties (IND-CCA2, existentially unforgeable),

Conclusion

Formal methods, including of course rewriting techniques, form a very powerful approach for analyzing security protocols

- Many automatic tools (ProVerif, Avispa, ...)
- Cryptographic guarantees

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Some current directions of research :

- Considering more equational theories (e.g. theories for e-voting protocols)
- Combining formal and cryptographic models
- Adding more complex structures for data (list, XML, ...)
- ...