Idempotent Generalization is Infinitary

David M. Cerna and Temur Kutsia

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Abstract

Let $I_S$ be an equational theory s.t. for each $f \in S$, $f(x, x) = x$. Such an equational theory is said to be idempotent. It is known that the anti-unification problem (AUP) $f(a, b) \equiv g(a, b)$ modulo $I_{\{f, g\}}$ admits infinitely many least-general generalizers (lggs) [1]. We show that, modulo $I_{\{f\}}$, $f(a, f(a, b)) \equiv f(b, f(a, b))$ admits infinitely many lggs.

Consider the incomparable lggs: $g_1 = f(f(x, f(x, b)), f(a, f(x, b)))$ and $g_2 = f(f(x, f(a, x)), f(f(x, b), f(a, b)))$. Note, $g_1 \{x \leftarrow a\} \approx g_2 \{x \leftarrow a\} \approx f(a, f(a, b))$ and $g_1 \{x \leftarrow b\} \approx g_2 \{x \leftarrow b\} \approx f(b, f(a, b))$. Let $S(i + 1) = f(g_1, f(g_2, S(i)))$ and $S(0) = g_1$. The set of incomparable lggs $\{S(n) | n \in \mathbb{N}\}$ is infinite.

References