Algebraic Methods in Kinematics
Homework 2

Let $G = (V, E)$ with labeling $\lambda : E \to \mathbb{C}$ be a rigid labelled graph with vertex set $V = \{1, \dots, n\}$. After some linear change of coordinates, the compatibilty condition for embeddings $\rho : V \to \mathbb{C}^2, v_i \to (\cup_i, \vee_i)$ is

$$(\cup_i - \cup_j)(\vee_i - \vee_j) - \lambda_{ij} = 0, (i, j) \in E,$$

together with 2 equations $\cup_1 = \vee_1 = \cup_2 + \vee_2 = 0$ that reduce equivalent realizations.
(a) Show that the bi-homogenization

$$(\cup_i - \cup_j)(\vee_i - \vee_j) - \lambda_{ij} \cup_0 \vee_0 = 0, (i, j) \in E,$$

$$\cup_1 = \vee_1 = \cup_2 \vee_0 + \cup_0 \vee_2 = 0$$

has infinitely many solutions in $\mathbb{P}^n \times \mathbb{P}^n$.
(b) If G is the triangle ($n = 3$), show that the multi-homogemization

$$(\cup_i \cap_j - \cap_i \cup_j)(\vee_i \wedge_j - \wedge_i \vee_j) - \lambda_{ij} \cap_i \cap_j \wedge_i \wedge_j = 0, (i, j) \in E,$$

$$\cup_1 = \vee_1 = \cup_2 \wedge_2 + \cap_2 \vee_2 = 0$$

has only finitely many solutions in $(\mathbb{P}^1)^{2n}$.