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A SIMPLIFIED PROOF OF THE CHARACTERIZATION THEOREM FOR GRÖBNER-BASES

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Abstract

In /2/ a certain type of bases ("Gröbner-Bases") for polynomial ideals has been introduced whose usefulness stems from the fact that a number of important computability problems in the theory of polynomial ideals are reducible to the construction of bases of this type. The key to an algorithmic construction of Gröbner-bases is a characterization theorem for Gröbner-bases whose proof in /2/is rather complex.

In this paper a simplified proof is given. The simplification is based on two new lemmas that are of some interest in themselves. The first lemma characterizes the congruence relation modulo a polynomial ideal as the reflexive-transitive closure of a particular reduction relation ("M-reduction") used in the definition of Gröbner-bases and its inverse. The second lemma is a lemma on general reduction relations, which allows to guarantee the Church-Rosser property under very weak assumptions.

1. Introduction

Gröbner-bases for polynomial ideals are defined as follows:

Definition:

A sequence F of polynomials from $K[x_1,\dots,x_n]$ is called a Gröbner-basis (for the ideal generated by F) iff

(G1) $g \in \text{Ideal}(F) \Rightarrow g \rhd_F 0$.

$\rhd_F$ is a certain reduction relation defined on $K[x_1,\dots,x_n]$ that depends on F. For the detailed definition of $\rhd_F$ and for the definition of all auxiliary notions as well as for the motivation for dealing with Gröbner-bases see /2/ and /3/.

The following characterization theorem provides an algorithmic test for the property of being a Gröbner-basis and allows an algorithmic construction of Gröbner-bases G that generate the same polynomial ideal as a given basis F, see /1/. In fact, the Knuth-Bendix algorithm and the extended Knuth-Bendix algorithm /6/ and the above algorithms have a very similar structure (see also section 4.).

Characterization Theorem /2/:

The following statements are equivalent:

(G1) F is a Gröbner-basis

(G2) for $1 \leq i < j \leq$ length of F: the S-polynomial of F_i and F_j $\rhd 0$

(G3) $(h \rhd \underset{\sim}{h_1}, h \rhd \underset{\sim}{h_2}) \Rightarrow h_1 = h_2$.

$\underset{\sim}{h_1}$ means that h_1 is in normalform with respect to the reduction relation $\rhd$. The notation $\rhd$ instead of $\rhd_F$ is used whenever F is clear from the context.

It should be mentioned that, in /2/, (G1) has been presented in the equivalent form

$$\underset{\sim}{g} \in \text{Ideal}(F) \Rightarrow g = 0$$

(see (G6) in /2/), and that (G3) is equivalent to:

(G3') $(h \rhd h_1, h \rhd h_2) \Rightarrow$
$\Rightarrow \bigvee_{g} (h_1 \rhd g, h_2 \rhd g)$

(the Church-Rosser property for $\rhd$).

The equivalence of (G3) and (G3') is a general result on noetherian relations, see /5/. In fact, various other equivalent formulations of (G1) and (G3) may be proven easily.

In /2/ a complex proof is necessary for obtaining ((G2) => (G3)) and an easier, but still tedious, proof establishes ((G3) => (G1)). ((G1) => (G2)) is immediate.

The above-mentioned algorithms are based on (G2), which only requires to reduce the "S-polynomials" of F_i and F_j (a certain type of "least common multiple" of F_i and F_j) for finitely many index pairs (i,j) in order to test a given F for being a Gröbner-basis. The complexity of the algorithms may be drastically decreased, /3/, by exploiting the following refinement of the characterization theorem:

<u>Theorem /7/:</u>

(G1) is equivalent to

(G8) for all $1 \leq i < j \leq$ length of F there exists a sequence $i=u_1, u_2, \ldots, u_k=j$ such that $H(u_1, \ldots, u_k) \leq_M H(i,j)$ and for all pairs (u_n, u_{n+1}) $(1 \leq n < k)$: the S-polynomial of F_{u_n} and $F_{u_{n+1}}$ $\rightarrow$ 0.

This means that it suffices to test whether all pairs (i,j) may be interconnected by certain "chains" of indices $u_1, \ldots, u_k$ such that the corresponding S-polynomials $SP(F_{u_1}, F_{u_2}), \ldots$ $\ldots, SP(F_{u_{k-1}}, F_{u_k})$ reduce to 0.

It is clear that ((G2) => (G8)). Thus, the interesting implications are ((G8) => (G3)) and ((G3) => (G1)).

In Sections 2 and 3 simplified proofs of ((G3) => (G1)) and ((G8) => (G3)), respectively, are given.

2. Proof of ((G3) => (G1))

This proof is based on the following new lemma:

<u>Lemma 1:</u>

Let F be an arbitrary sequence of polynomials (not necessarily a Gröbner-basis), then $f \equiv_F g$ <=> f vvv g.
                                                                   F

Here $\equiv_F$ is the congruence relation modulo the ideal generated by F, i.e.

$$f \equiv_F g \; : \Leftrightarrow \; f = g + \sum_{i=1}^{L(F)} h_i . F_i$$

for certain polynomials $h_1, \ldots, h_{L(F)}$
(L(F)...length of F).

vvv denotes the reflexive-transitive
 F
closure of the reduction relation $>_F$ and its inverse, i.e.

f vvv g : <=> there are polynomials $h_1, \ldots, h_k$ such that $f=h_1, g=h_k$ and for $1 \leq i < k$
   F
$h_i >_F h_{i+1}$ or
$h_{i+1} >_F h_i$.

We again use $\equiv$ and vvv instead of $\equiv_F$ and vvv , resp., if F is clear from
          F
the context.

Lemma 1 so far has escaped our attention, although it turns out to be an easy consequence of property (R1) in /2/. Lemma 1 establishes an easy connection between those formulations of the concept of Gröbner-basis using the ideal-theoretic notion of congruence and those using the notion of M-reduction. The reader is advised to carefully examine the definition of the relation $>_F$ in order to see, why the lemma is non-trivial.

<u>Proof of Lemma 1:</u>

<=: Immediate from the definitions (see (E5) in /2/).

=>: We show by induction on m that

$$f = g + \sum_{j=1}^{m} a_j . t_j . F_{i_j}$$

$(a_1, \ldots, a_m \epsilon K; t_1, \ldots, t_m$ terms)

implies

f vvv g.

From this (f ≡ g => f vvv g) may be concluded because if f ≡ g, then

$$f = g + \sum_{j=1}^{m} a_j . t_j . F_{i_j}$$

for certain $a_1, \ldots, a_m \epsilon K$ and terms $t_1, \ldots, t_m$.

m=1: Let $f = g + a_1 . t_1 . F_{i_1}$. It is clear that
$a_1 . t_1 . F_{i_1} >^1 0$
(subtract $a_1 . t_1 . F_{i_1}$ from $a_1 . t_1 . F_{i_1}$:

this is an admissible $>^1$-step!).
Then, by property (R1) of Lemma 2.4. in /2/,

$$f = g + a_1 . t_1 . F_{i_1} \overset{succ}{\nabla} g + 0 = g$$

(i.e. f and g have a common successor),

Of course $f \overset{succ}{\nabla} g$ is a special case of f vvv g.

m>1: Let $f = g + \sum_{j=1}^{m} a_j . t_j . F_{i_j} = g + a_1 . t_1 . F_{i_1} + \sum_{j=2}^{m} a_j . t_j . F_{i_j}$.

Then by induction hypothesis:

$$f \text{ vvv } g + a_1 . t_1 . F_{i_1}$$

and, as in the case m=1,

$$g + a_1 . t_1 . F_{i_1} \overset{succ}{\nabla} g$$

and therefore f vvv g.

Proof of ((G3) => (G1)) by Lemma 1:

The proof of ((G3) => (G1)) is easy now. Assume (G3). Then

$g \in \text{Ideal}(F) \Rightarrow g \equiv 0 \Rightarrow g \text{ vvv } 0 \Rightarrow$
$\Rightarrow$ (from (G3')) $g \overset{succ}{\nabla} 0 \Rightarrow$
$\Rightarrow$ (0 is in normalform!) $g = 0$.

(g vvv f => $g \overset{succ}{\nabla} f$ is a wellknown consequence of the Church-Rosser property (G3')).

We note that a direct proof of ((G1) => => (G3)) without the intermediate (G2) is easy, too:

$h > \underset{\sim}{h_1}, h > \underset{\sim}{h_2} \Rightarrow \underset{\sim}{h_1} \equiv h \equiv \underset{\sim}{h_2}$
$\Rightarrow \underset{\sim}{h_1 - h_2} \equiv 0 \Rightarrow$
$\Rightarrow$ (from (G1)) $\underset{\sim}{h_1 - h_2} > 0 \Rightarrow$
$\Rightarrow$ ($h_1 - h_2$ is in normalform!)
$h_1 - h_2 = 0$
$\Rightarrow h_1 = h_2$.

Similarly, using Lemma 1, many variants of (G1) and (G3) can easily be proven equivalent. Thus the attention is lead to the central point of the characterization theorem asserting that the algorithmic properties (G2) and (G8) resp. are sufficient criteria for Gröbner-bases.

3. Proof of ((G8) => (G3))

In order to make the presentation more readable and to single out the essential points of the simplification a simplified proof of ((G2) => (G3)) is presented first. Of course, logically, this proof will be superseded by the subsequent proof of ((G8) => (G3)).

The essential simplification in the proof of ((G2) => (G3)) consists in the application of a general lemma on noetherian reduction relations (see for instance /5/) showing that a certain "local" Church-Rosser property implies the global Church-Rosser property.

Analogously, a new lemma on arbitrary reduction relations, showing that the Church-Rosser property may be asserted under weaker assumptions, allows a simplification in the structure of the proof of ((G8) => (G3)). In essence, the new general lemma arises from the lemma in /5/ by a refinement analogous to that by which condition (G8) arises from (G2). Thus, the results presented in this section may also be viewed as a means of exploiting the refined method developed in /7/and /3/ for polynomial reductions for the case of arbitrary noetherian reduction relations. This method could prove useful, for instance, in various term algebras in which the Knuth-Bendix algorithm is applied.

In order to make the presentation selfcontained the following notations and results are resumed from /5/.

Let M be an arbitrary set and $\rightarrow$ a reduction relation on M. $\rightarrow^+$ denotes the transitive closure of , $\rightarrow^*$ denotes the transitive-reflexive closure of $\rightarrow$.

Definition:

$\rightarrow$ is noetherian iff there is no infinite sequence $x_1 \rightarrow x_2 \rightarrow \dots \rightarrow x_n \rightarrow \dots$
$\nabla^+(x) := \{z \in M \mid x \rightarrow^+ z\}$.
$x \nabla y :\Leftrightarrow \bigvee_z (x \rightarrow^* z \wedge y \rightarrow^* z)$.
$x \rightarrow^* \underset{\sim}{y} :\Leftrightarrow x \rightarrow^* y \wedge \neg(\bigvee_z y \rightarrow z)$
(y is in $\rightarrow$-normalform).

Definition:

$\rightarrow$ is CR (Church-Rosser) : <===>
: <===> $\bigwedge_{x,y,z} (x \rightarrow^* y, x \rightarrow^* z \Rightarrow y \nabla z)$.

Definition:

$\rightarrow$ is locally CR : <===>
: <===> $\bigwedge_{x,y,z} (x \rightarrow y, x \rightarrow z \Rightarrow y \nabla z)$.

Lemma 2, /5/:

A noetherian reduction relation is CR iff it is locally CR.

In the sequel the following elementary properties of polynomial M-reduction are used (Proofs may be found in /2/).

(E1) $f \underset{Hterm(f)}{\overset{1}{>}} g,\ Hterm(f)\ {}_T{>}\ Hterm(h)$
$\Rightarrow f+h \overset{1}{>} g+h.$

(E2) $g \overset{1}{>} h,\ \bigwedge_t (Coef(t,f) \neq 0 \Rightarrow t_T{>}Hterm(g))$
$\Rightarrow f+g \overset{1}{>} f+h.$

(E3) $f>g \Rightarrow a.t.f>a.t.g$ ($a \in K$, t a term).

(E4) $f \overset{1}{>} g \Rightarrow f+h \overset{succ}{\nabla} g+h.$

(E5) $f-g > 0 \Rightarrow f \overset{succ}{\nabla} g.$

(E6) $>$ is a noetherian reduction relation.

Proof of ((G2) => (G3)) by Lemma 2:

Sketch: We show that if $>$ satisfies (G2) then $>$ is locally CR. The assertion then follows from Lemma 2 and (E6).

Details:

Assume (G2), i.e.

(i) $\bigwedge_{1 \leq i<j \leq L(F)} SP(F_i,F_j)>0.$

We shall show that

(ii) $\bigwedge_{f,g,h} (f \overset{1}{>} g, f \overset{1}{>} h \Rightarrow g \overset{succ}{\nabla} h).$

Let f,g,h,t,s,i,j be such that

(iii) $f \underset{t,i}{\overset{1}{>}} g,\ f \underset{s,j}{\overset{1}{>}} h.$

Without loss of generality we may assume $s \leq_T t$. We distinguish the cases $s<_T t$ and $s=t$.

Case 1: $s<_T t$

There are polynomials f_1,f_2,g_1,h_1 such that $f=f_1+a.t+f_2$ and

(iv) $\bigwedge_{t'} (Coef(t',f_1) \neq 0 \Rightarrow t'_T{>}t),$

(v) $t_T{>}Hterm(f_2),$

(vi) $a.t \underset{t,i}{\overset{1}{>}} g_1,$

(vii) $f_2 \underset{s,j}{\overset{1}{>}} h_1.$

From (vi), (vii), (E1) and (E2) we easily deduce

(viii) $g=f_1+g_1+f_2,$

(ix) $h=f_1+a.t+h_1.$

Furthermore (vi), (E1), (E2) yield

(x) $h \underset{t,i}{\overset{1}{>}} f_1+g_1+h_1.$

(vii) and (E4) imply

(xi) $g=f_1+g_1+f_2 \overset{succ}{\nabla} f_1+g_1+h_1.$

Thus, from (x) and (xi)

(xii) $g \overset{succ}{\nabla} h.$

Case 2: $s=t$

Then g and h are such that

(xiii) $g=f-\frac{Coef(t,f)}{Hcoef(F_i)} \cdot \frac{t}{Hterm(F_i)} \cdot F_i,$

(xiv) $h=f-\frac{Coef(t,f)}{Hcoef(F_j)} \cdot \frac{t}{Hterm(F_j)} \cdot F_j.$

Let the term t' be such that

(xv) $t=t'.Lcm(Hterm(F_i),Hterm(F_j)).$

In order to show $g \overset{succ}{\nabla} h$ we observe that

(xvi) $g-h=\frac{Coef(t,f)}{Hcoef(F_i).Hcoef(F_j)} \cdot$
$\cdot (Hcoef(F_i) \cdot \frac{t}{Hterm(F_j)} \cdot F_j -$
$-Hcoef(F_j) \cdot \frac{t}{Hterm(F_i)} \cdot F_i) =$
$= \frac{Coef(t,f)}{Hcoef(F_i).Hcoef(F_j)} \cdot t' \cdot$
$\cdot SP(F_j,F_i).$

From (i), (xvi), and (E3) we deduce

(xvii) $g-h > 0.$

(E5), then, yields the assertion (xii).

We now present the above-mentioned refinement of Lemma 2.

Definition:

Let $\rightarrow$ be a reduction relation on M. $\rightarrow$ is locally pseudo-CR iff

$\bigwedge_{x,y,z} (x \rightarrow y, x \rightarrow z \Rightarrow \bigvee_{u_1,\ldots,u_n} (y=u_1, u_n=z,$
$\bigwedge_{1 \leq k<n} (x \overset{+}{\rightarrow} u_k, u_k \nabla u_{k+1}))).$

Lemma 3:

A noetherian reduction relation is CR iff it is locally pseudo-CR.

Proof:

=>: trivial.

<=: Assume $\rightarrow$ is a noetherian reduction relation and is locally pseudo-CR, i.e.

(i) $\bigwedge_{x,y,z} (x \to y, x \to z \Rightarrow \bigvee_{u_1,\dots,u_n} (y=u_1, u_n=z, \bigwedge_{1\le k<n} (x \overset{+}{\to} u_k, u_k \nabla u_{k+1})))$.

We show

(ii) $\bigwedge_{x,y,z} (x \overset{*}{\to} y, x \overset{*}{\to} z \Rightarrow y=z)$.

(a variant of the CR property).
We give a proof by noetherian induction:

Induction hypothesis: for a fixed $\hat{x} \in M$:

(iii) $\bigwedge_{x \in \nabla^+(\hat{x})} \bigwedge_{y,z} (x \overset{*}{\to} y, x \overset{*}{\to} z \Rightarrow y=z)$.

We shall show:

(iv) $\bigwedge_{y,z} (\hat{x} \overset{*}{\to} y, \hat{x} \overset{*}{\to} z \Rightarrow y=z)$.

Let y,z be such that

(v) $\hat{x} \overset{*}{\to} y, \hat{x} \overset{*}{\to} z$.

We distinguish the following cases:

Case 1: $\hat{x}=y \vee \hat{x}=z$: trivial.

Case 2: $\hat{x} \neq y$, $\hat{x} \neq z$.

Then there exist y_1, z_1 such that

(vi) $\hat{x} \to y_1 \overset{*}{\to} y$,

(vii) $\hat{x} \to z_1 \overset{*}{\to} z$.

By applying (i) to $\hat{x}, y_1, z_1$ we get

$u_1, \dots, u_n$ such that

(viii) $y_1=u_1, u_n=z_1, \bigwedge_{1\le k<n} (\hat{x} \overset{+}{\to} u_k, u_k \nabla u_{k+1})$.

Now let $v_1, \dots, v_{n-1}$ be such that

(ix) $\bigwedge_{1\le k<n} (u_k \overset{*}{\to} v_k, u_{k+1} \overset{*}{\to} v_k)$.

Then

(x) $u_1 \overset{*}{\to} y, u_1 \overset{*}{\to} v_1$,

(xi) $\bigwedge_{1<k<n} (u_k \overset{*}{\to} v_k, u_k \overset{*}{\to} v_{k-1})$,

(xii) $u_n \overset{*}{\to} v_{n-1}, u_n \overset{*}{\to} z$.

From (x), (xi), (xii) and induction hypothesis (iii) we obtain (using (viii)) (xiii) $y=v_1=\dots=v_{n-1}=z$, which concludes the proof.

Proof of ((G8) => (G3)) by applying Lemma 3:

Sketch: We show that if > satisfies (G8) then > is locally pseudo-CR. The assertion then follows from Lemma 3 and (E6).

Details:

Assume (G8), i.e.

(i) $\bigwedge_{1\le i<j\le L(F)} \bigvee_{1\le u_1,\dots,u_n\le L(F)} (i=u_1, u_n=j, H_F(u_1,\dots,u_n) \le_M H_F(i,j), \bigwedge_{1\le k<n} SP(F_{u_k}, F_{u_{k+1}}) > 0)$.

We shall show

(ii) $\bigwedge_{f,g,h} (f >^1 h, f >^1 g \Rightarrow \bigvee_{f_{u_1},\dots,f_{u_n}} (g=f_{u_1}, f_{u_n}=h, \bigwedge_{1\le k<n} (f >^1 f_{u_k}, f_{u_k} \overset{succ}{\nabla} f_{u_{k+1}})))$.

Let f,g,h,t,s be such that

(iii) $f \underset{t,i}{>}^1 g$, $f \underset{s,j}{>}^1 h$.

Without loss of generality we may assume $s \le_T t$. We distinguish again the cases

$s=t$ and $s <_T t$.

Case 1: $s <_T t$.

Analogous to Case 1 in the proof of ((G2) => (G3)).

Case 2: s=t.

We can write f in the following form: $f=f_1+a.t+f_2$ with

(iv) $\bigwedge_{t'} (Coef(t', f_1) \neq 0 \Rightarrow t' >_T t)$,

(v) $t >_T Hterm(f_2)$.

Without loss of generality we may assume i<j.
Take suitable $u_1, \dots, u_n$ such that (i) is valid.
We know from (i) that $H_F(u_1,\dots,u_n) \le_M$

$\le_M H_F(i,j)$. Therefore

(vi) $\bigwedge_{1\le k\le n}$ t is a multiple of $H_F(u_k)$.

Thus, by definition of M-reduction

(vii) $\bigwedge_{1\le k\le n} a.t \underset{u_k}{>}^1 g_{u_k} := a.t - \frac{a}{Hcoef(F_{u_k})} \cdot \frac{t}{Hterm(F_{u_k})} \cdot F_{u_k}$.

From (v), (E1), (iv), (E2) we may conclude

(viii) $\bigwedge_{1\le k\le n} f=f_1+a.t+f_2 >^1 f_1+g_{u_k}+f_2$.

Now let $f_{u_k} := f_1+g_{u_k}+f_2$, $1\le k\le n$.

We know from (i), (iii), (vii) and (viii):
$g=f_{u_1}, f_{u_n}=h, \bigwedge_{1 \leq k \leq n} f >^1 f_{u_k}$.

We shall show

(ix) $\bigwedge_{1 \leq k<n} f_{u_k} \overset{succ}{\nabla} f_{u_{k+1}}$.

An easy calculation shows (see also the above proof of ((G2) => (G3))

(x) $$\bigwedge_{1 \leq k<n} \bigvee_{b_k, t_k} f_{u_{k+1}} - f_{u_k} = (f_1+g_{u_{k+1}}+f_2) - (f_1+g_{u_k}+f_2) = g_{u_{k+1}} - g_{u_k} = b_k \cdot t_k \cdot SP(F_{u_k}, F_{u_{k+1}}).$$

Therefore by (E3) and (i)

(xi) $$\bigwedge_{1 \leq k<n} f_{u_{k+1}} - f_{u_k} \rightarrow 0.$$

Thus, by (E5), we obtain (ix) which completes the proof.

4. Conclusion

Loos /8/ conjectures that the algorithm in /1/ based on (G2) (not the refinement in /3/ based on (G8)) may be viewed as a special case of the Knuth-Bendix algorithm by a suitable interpretation of the notion of term in the Knuth-Bendix algorithm. We remark that (G2) may be replaced by

(G2') $\bigwedge_{1 \leq i<j \leq L(F)} g^i_{i,j} \overset{succ}{\nabla} g^j_{i,j}$, where

$$g^i_{i,j} := H_F(i,j) \frac{1}{Hcoef(F_i)} \cdot \frac{H_F(i,j)}{H_F(i)} \cdot F_i,$$

$$g^j_{i,j} := H_F(i,j) \frac{1}{Hcoef(F_j)} \cdot \frac{H_F(i,j)}{H_F(j)} \cdot F_j.$$

(see /4/). This means that the check, whether the difference of $g^i_{i,j}$ and $g^j_{i,j}$ (=S-polynomial of F_i and F_j) may be replaced by the check, whether $g^j_{i,j}$ and $g^i_{i,j}$ have a common successor.

This observation and the deduction of both the Knuth-Bendix criterion and our (G2') from the same Lemma 2 (compare the presentation in /5/) shows that the conjecture is reasonable.

It should be also noted that property (E5) (=(R3) in /2/) can be eliminated from the proof of ((G2') => (G3)), such that (E4) (=(R1) in /2/) seems to be the property of $>^1$ that is central to the above proofs. This gives some hints how to characterize those domains to which our own algorithm may be generalized.

References

/1/ B. Buchberger, Ph.D.Thesis, Univ. Innsbruck, 1965 (see also Aequationes mathematicae, Vol.4/3, pp.374-383, 1970).

/2/ B. Buchberger, A Theoretical Basis for the Reduction of Polynomials to Canonical Forms, ACM-SIGSAM Bulletin 39, August 1976, p.19-29.

/3/ B. Buchberger, A Criterion for Detecting Unnecessary Reductions in the Construction of Gröbner-Bases, in: Proc. EUROSAM 79, Marseille, (Lecture Notes in Computer Science, Vol. 72), pp. 3-21, 1979.

/4/ B. Buchberger, F. Winkler, Miscellaneo Results on the Construction of Gröbner-Bases for Polynomial Ideals, Bericht Nr. 137, Institut für Mathematik, Universität Linz, 1979.

/5/ G. Huet, Confluent Reductions: Abstract Properties and Applications to Term Rewriting Systems, 18th IEEE Symp. on Found.Comp.Scie, 30-45, 1977.

/6/ D. Knuth, P. Bendix, Simple word problems in universal algebras, Computational problems in abstract algebra, Ed. J. Leech, Pergamon Press, pp. 263-297, 1970.

/7/ C. Kollreider, B. Buchberger, An Improved Algorithmic Construction of Gröbner-Bases for Polynomial Ideals, Bericht Nr. 110, Institut für Mathematik, Universität Linz, 1978.

/8/ R. Loos, personal communication, 1979.

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