to be prepared for 12.10.2023

Exercise 1. Consider the integers $a = 215712,\ b = 739914$. Determine the gcd of a and b and integers s, t such that $\gcd(a, b) = sa + tb$.

Exercise 2. Compute the GCD of

$$\begin{aligned} f(x) &= 6x^5 + 2x^4 - 19x^3 - 6x^2 + 15x + 9 \\ g(x) &= 5x^4 - 4x^3 + 2x^2 - 2x - 2\ . \end{aligned}$$

Exercise 3. Demonstrate that a finite integral domain is necessarily a field.

Exercise 4. Write an algorithm that inverts unit elements in $\mathbb{Z}/n\mathbb{Z}$ ($n \in \mathbb{N}$).

Exercise 5. Consider the polynomials $f_1, \ldots, f_m$ in the ring $K[x_1, \ldots, x_n]$. Prove that the set

$$\left\{ \sum_{j=1}^{m} h_j f_j \mid \forall_{1 \le j \le m}\, h_j \in K[x_1, \ldots, x_n] \right\}$$

is the smallest ideal in $K[x_1, \ldots, x_n]$ that contains the set $\{f_1, \ldots, f_m\}$.

Exercise 6. We can check that the polynomial ring $K[x]$ over a field K is an integral domain by considering the degree function.

1. Prove, by a similar argument, that also the ring $K[[x]]$ of formal power series is an integral domain. Moreover show that K[[x]] is an Euclidean domain by introducing an appropriate function and verifying the required properties.
2. Show that $K[[x]]$ is a local ring.