Sheet 10

Discussion on **19 January, 2023**

Exercise 1 Consider the functions

$$f(x) = \sum_{n=0}^{\infty} a(n)x^n,$$
$$g(x) = 3x + x^2.$$

Compute the first four coefficients of $g \circ f(x)$. Can arbitrarily many coefficients be so computed? Is there more than one solution?

Exercise 2 Compute the multiplicative inverses of the following:

$$\sum_{n=0}^{\infty}(-1)^n \frac{x^n}{5^n}, \quad x^2 - 6x + 9.$$

Exercise 3 Consider the function

$$f(x) = \sin(x).$$

Can we use the formula in Theorem 7.14 to compute $\left(f^{-1}\right)'(1/2)$? If so, compute it.

Exercise 4 Suppose that functions $f, g : (a, b) \to \mathbb{R}$ are differentiable on the interval (a, b). Prove Theorem 7.7(1).

Exercise 5 Suppose that functions $f, g : (a, b) \to \mathbb{R}$ are differentiable on the interval (a, b), and that $g(x) \neq 0$ for $x \in (a, b)$. Prove Theorem 7.7(3) *Hint: Use Theorem 7.7(2).*

Exercise 6 Compute the derivatives of the following functions:

a) $f(x) = \sqrt{\frac{x^3+x}{x^2+1}}$

b) $g(x) = \frac{x^2 \sin(x)}{(1+x)\ln(x)}$

c) $h(x) = \cos\left(\exp\left(\sqrt{x}\right)\right)$

Exercise 7 Compute the derivatives of the following functions:

a) $f(x) = e^{\cos(x)} + \ln(x)$

b) $g(x) = \arcsin(x)$

c) $h(x) = \tan(x)$

Exercise 8 Consider the differential equation

$$xf''(x) - 5f'(x) + f(x) = 0.$$

Suppose that we can represent a solution function $f(x)$ as a power series in x, i.e., $f(x) = \sum_{n=0}^{\infty} a_n x^n$. Determine a formula for the coefficients a_n of x^n; this formula might be recursive—that is, a_n may depend on previous coefficients $a_{n-1}, a_{n-2}, \ldots$.