Analysis Exercises **Wintersemester 2021**
https://www.risc.jku.at/education/courses/ws2021/mathematik1/

Sheet 4

Discussion on **Nov. 4, 2021**

Exercise 1 Let $\mathcal{A}$ be an ordered set, and let $\mathcal{B} \subseteq \mathcal{A}$. The set $\mathcal{B}$ is said to be *dense subset* of $\mathcal{A}$ if, for any $a \in \mathcal{A} \setminus B$ there exists some $b \in \mathcal{B}$ arbitrarily close to a.

a) Is $\mathbb{Z}$ dense in $\mathbb{R}$?

b) Is $\mathbb{Q}$ dense in $\mathbb{R}$?

c) Is $\{n\pi : n \in \mathbb{Z}\}$ dense in $\mathbb{R}$?

Justify your answers.

Exercise 2 Prove the first part of Lemma 1.40: For any two real numbers $x, y \in \mathbb{R}$, we have:

$$|x + y| \leq |x| + |y|. \tag{1}$$

Exercise 3 Prove Lemma 1.42.

Exercise 4 Let a and b be computable real numbers. Using Definition 1.43, show that $a + b$ is also computable.

Exercise 5 Prove the Pythagorean Theorem (Theorem 2.6). *Hint: Consult Euclid.*

Exercise 6 Prove Lemma 2.7.

Exercise 7 Which of these functions are injective, surjective, or bijective? Pay close attention to the domain and range.

a) $f : \mathbb{R} \setminus \{0\} \longrightarrow \mathbb{R} \setminus \{0\}, x \mapsto \frac{1}{x}$.

b) $g : (-\infty, 0] \longrightarrow [0, \infty), x \mapsto x^2 + 2$.

c) $g : (-\infty, 0] \longrightarrow [2, \infty), x \mapsto x^2 + 2$.

d) $g : (-\infty, 1] \longrightarrow [2, \infty), x \mapsto x^2 + 2$.

Exercise 8 Suppose that I take a meterstick and place it at a right angle to the ground in Linz. It casts a shadow that is 17.63 cm in length.

a) Calculate the angles for the resulting right triangle.

b) At the same time, my friend tries the same experiment in Rome, 727 km away. She gets a shadow length of 6.06 cm. Calculate the angles for her triangle.

c) Incidentally, what is the circumference of the Earth?