Due date: 23.11.2021

We will discuss the content of Exercise sheet 4 as well.

Exercise 18. Consider non-constant polynomials $f, g \in I[x]$ over an integral domain I such that $m = \deg(f)$ and $n = \deg(g)$. Prove the subsequent resultant properties:

(a) $\mathrm{res}_x(f, g) = (-1)^{mn}\mathrm{res}_x(g, f)$.

(b) If $\lambda, \mu \in I^*$, then $\mathrm{res}_x(\lambda f, \mu g) = \lambda^n \mu^m \mathrm{res}_x(f, g)$.

(c) Let $p = x^5 - 3x^4 - 2x^3 + 3x^2 + 7x + 6$ and $q = x^4 + x^2 + 1$. Construct the Sylvester matrix $\mathrm{Syl}_x(p, q)$ and compute the resultant $\mathrm{res}_x(p, q)$. What does the resultant tell you about common factors of p and q in $\mathbb{Q}[x]$?

Exercise 19. Let $f, g \in K[x]$ be non-constant polynomials over a field K. Perform division with remainder of f by g, i.e. $f = qg + r$, where $q, r \in K[x]$ and $r = 0$ or $\deg(r) < \deg(g)$. Show that if r is non-constant, then

$$\mathrm{res}_x(f, g) = (-1)^{m\,n}\,\mathrm{lc}(g)^{m-o}\,\mathrm{res}_x(g, r),$$

where $m = \deg(f)$, $n = \deg(g)$, and $o = \deg(r)$.