

# Computer Algebra

Lecture Notes

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# Contents

References	
1. What is Computer Algebra ?	1
2. Greatest common divisors of polynomials	5
2.1 The Euclidean algorithm	5
2.2 A modular GCD algorithm	10
2.3 Squarefree factorization	16
3. Resultants	19
4. Gröbner bases	27
4.1. Introduction: From Gauss to Gröbner	27
4.2. Basic theory of Gröbner bases	31
4.3. Solving problems in polynomial ideal theory by Gröbner bases	45
4.4. Gröbner bases at work in robotics	61
5. Factorization	65
5.1. Factorization over finite fields	65
5.2. Factorization over the integers	69
5.3. Factorization over algebraic extension fields	75
6. Appendix: Arithmetic in basic domains	79

The material in these lecture notes is largely taken from

F. Winkler, “Polynomial Algorithms in Computer Algebra”, Springer-Verlag Wien  
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where also proofs of the theorems can be found.

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