Due date: 13.10.2020

The exercises are meant as an invitation to become acquainted with a computer algebra system (CAS) of your choice. Exercise sheets contain problems which have to be solved on a computer from time to time. Furthermore, the project will contain explicit programming tasks so it is a good idea to familiarize yourself with a CAS early on.

The following selection lists some of the most popular and user-friendly systems.

1. Wolfram Mathematica (https://www.wolfram.com/mathematica/)
2. Maple (https://www.maplesoft.com/products/maple/)
3. SageMath (https://www.sagemath.org/): Downloadable for free at http://www.sagemath.org/download.html

1. Exercise

Given the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 1 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 2 & 3 \\ 0 & 0 & 0 & 3 & 4 \end{bmatrix}.$$

Compute all solutions of the linear system $A \cdot [x_1, x_2, x_3, x_4, x_5]^T = [1, 2, 3, 4, 5]^T$ with the aid of a CAS.

2. Exercise

Consider the polynomial $f = x^5 - x^4 + x^3 - x^2 + x - 2$. Use a CAS to perform the following tasks:

1. Compute the roots of f numerically. You have influence on floating point precision if you want to.
2. Generate an image of the graph of the polynomial function $F : [a, b] \to \mathbb{R},\ x \mapsto f(x)$. Choose the boundaries a and b of the interval in such a way that you can "see" the real roots of f.
3. Compute the roots of f symbolically. What output does your CAS generate?
4. Compute the roots of the polynomial $g = 2x^2 + 2x^3 + 2x^4 + x^5 - x^6 + 3x + 1$.

3. Exercise

Use a CAS to compute greatest common divisors (GCDs) in different domains.

1. Compute the integer GCD of the numbers $a = 215712$ and $b = 739914$. Is the number 48510 contained in the ideal generated by a and b?
2. Compute the polynomial GCD in $\mathbb{Q}[x]$ of $f = 6\,x^5 + 2\,x^4 - 19\,x^3 - 6\,x^2 + 15\,x + 9$ and $g = 5\,x^4 - 4\,x^3 + 2\,x^2 - 2\,x - 2$. Is the ideal generated by f and g the whole ring $\mathbb{Q}[x]$?

   Hint: Both ideals are generated by the corresponding GCDs.

4. Exercise

Find a rational parametrization of the unit circle $x^2 + y^2 = 1$.