

Logic 1, WS 2016. Homework 7, given Nov 24, due Dec 1.

1. Compute the truth value: $\langle P[x] \Rightarrow P[f[x]] \rangle_{\{x \rightarrow 1\}}^I$ with respect to the interpretation $I : \{D_I = \{0, 1\}, a_I = 1, f_I = \{0 \rightarrow 1, 1 \rightarrow 0\}, P_I = \{0 \rightarrow T, 1 \rightarrow F\}\}$.
2. Prove by definition $\neg \exists_x \varphi \equiv \forall_x \neg \varphi$.
3. Show by a counterexample that $\exists_x (\varphi \wedge \psi)$ is not equivalent to $(\exists_x \varphi) \wedge (\exists_x \psi)$.
4. Prove by reducing to normal form: $(\exists_x P[x]) \Rightarrow Q \equiv \forall_x (P[x] \Rightarrow Q)$.
5. Show that if $\forall_x P[x, f[x]]$ is satisfiable, then $\forall_{x,y} P[x, y]$ is satisfiable.