to be prepared for 29.11.2016

Exercise 31. $R[X] = R[x_1, \ldots, x_n]$ denotes the polynomial ring in n indeterminates over a commutative ring with 1. Any admissible ordering $<$ on the monoid of power products $[X]$ induces a strict partial ordering $<<$ on $R[X]$, the `induced ordering`, in the following way:

$f << g$ iff $f = 0$ and $g \neq 0$ or
$f \neq 0$, $g \neq 0$ and $\mathrm{lpp}(f) < \mathrm{lpp}(g)$ or
$f \neq 0$, $g \neq 0$, $\mathrm{lpp}(f) = \mathrm{lpp}(g)$ and $\mathrm{red}(f) << \mathrm{red}(g)$.

Prove that $<<$ (or actually $>>$) is a Noetherian partial order on $R[X]$.

Exercise 32. Let $<$ be an admissible ordering on $[X]$ and $F \subseteq K[X]$. Prove that the reduction $\longrightarrow_F$ is a Noetherian relation on $R[X]$.

Exercise 33. Prove the following theorem (Theorem 2.3.14 (a,b,c)):

Let $c \in K \setminus 0$, $s \in [X]$, $F \subseteq K[X]$, $g_1, g_2 \in K[X]$.

(a) $\longrightarrow_F \subseteq >>$,

(b) $\longrightarrow_F$ is Noetherian,

(c) if $g_1 \longrightarrow_F g_2$ then $csg_1 \longrightarrow_F csg_2$,

Exercise 34. Let $F \subseteq K[X]$, $g_1, g_2, h \in K[X]$. Prove the following statement (Theorem 2.3.14 (d)):

If $g_1 \longrightarrow_F g_2$ then $g_1 + h \downarrow_F^* g_2 + h$.

Exercise 35. Verify the statement of Lemma 2.3.14 (d) by the following example: $R = K[x, y]$, $F = \{x^2y^2 + y - 1,\ x^2y + x\}$, $g_1 = x^5y^5$, $h = x^3y^3$.