

to be prepared for 25.10.2016

Exercise 11. Let D be a Euclidean domain. Prove the following lemma.

1. Let $m_1, \dots, m_n \in D^*$ be pairwise relatively prime and let $M = \prod_{i=1}^{n-1} m_i$. Then m_n and M are relatively prime.
2. Let $r, r' \in D$, and $m_1, m_2 \in D^*$ be relatively prime. Then $r \equiv r' \pmod{m_1}$ and $r \equiv r' \pmod{m_2}$ if and only if $r \equiv r' \pmod{m_1 m_2}$.

Exercise 12. Consider the two polynomials over \mathbb{Z}

$$\begin{aligned} f(x) &= 6x^5 + 2x^4 - 19x^3 - 6x^2 + 15x + 9 \\ g(x) &= 5x^4 - 4x^3 + 2x^2 - 2x - 2. \end{aligned}$$

Compute $\gcd(f(x), g(x))$ by the modular algorithm.

Exercise 13. Given the polynomials

$$\begin{aligned} f(x) &= x^7 - 3x^5 - 2x^4 + 13x^3 - 15x^2 + 7x - 1, \\ g(x) &= x^6 - 9x^5 + 18x^4 - 13x^3 + 2x^2 + 2x - 1 \end{aligned}$$

compute their $\gcd h \in \mathbb{Z}[x]$. Check whether the integer factors of the resultant of f/h and g/h are unlucky primes in the modular approach to gcd computation.

Exercise 14. Consider the bivariate polynomials

$$\begin{aligned} f(x, y) &= x^2y^3 - 5xy^3 + 6y^3 - 6xy^2 + 18y^2 + 2xy - 4y - 12, \\ g(x, y) &= x^2y^3 + 3xy^3 - 10y^3 - 6xy^2 - 30y^2 - 4xy + 8y + 24. \end{aligned}$$

Compute the gcd of f and g by the modular algorithm. Take care of leading coefficients.

Exercise 15. Compute the resultant of the two polynomials

$$\begin{aligned} f &= 3x^2 + x - 1 \\ g &= x^3 - x^2 + 2x - 3. \end{aligned}$$