to be prepared for 22.11.2016

Exercise 26. Apply the modular algorithm for computing the resultant $\mathrm{res}_y(a, b)$ of the polynomials

$$\begin{array}{rcl} a(x,y) & = & xy^2 - x^3y - 2x^2y + xy + 2x^4 - 2x^2 \\ b(x,y) & = & 2x^2y^2 - 4x^3y + 4x^4. \end{array}$$

Exercise 27. According to Theorem 4.3.3 a solution of $\mathrm{res}_{x_r}(a, b) = 0$ can be extended to a common zero of a and b, in case the leading coefficient of a or b is a nonzero constant. This can be achieved by a suitable change of coordinates. Work out an algorithm for solving systems of algebraic equations by resultants along these lines.

Exercise 28. Let $<$ be an admissible ordering on $[X]$. Prove the following statements.

1. If $s, t \in [X]$ and s divides t then $s \leq t$.
2. $<$ (or actually $>$) is Noetherian, i.e. there are no infnite chains of the form $t_0 > t_1 > t_2 > \cdots$, and consequently every subset of $[X]$ has a smallest element.

Exercise 29. The graduated reverse lexicographic ordering on power products of $x_1, \ldots, x_n$ $<_{\mathrm{grlex}}$ is defined by

$$\begin{array}{lll} s <_{\mathrm{grlex}} t \quad \text{iff} & \deg(s) < \deg(t) & \text{or} \\ & \deg(s) = \deg(t) & \text{and} \quad t <_{\mathrm{lex},\pi} s; \end{array}$$

where π is the permutation on n letters given by $\pi(j) = n - j + 1$ and $<_{\mathrm{lex},\pi}$ is the lexicographic order wrto. π. Prove that $<_{\mathrm{grlex}}$ is an admissible ordering.

Exercise 30. Let $<_1$ be an admissible ordering on $X_1 = [x_1, \ldots, x_i]$ and $<_2$ an admissible ordering on $X_2 = [x_{i+1}, \ldots, x_n]$. Show that the product ordering $<_{prod,i,<_1,<_2}$ on $X = [x_1, \ldots, x_n]$ is an admissible ordering.