to be prepared for 15.11.2016

Exercise 21. $f, g \in k[x]$, divide f by g, $f = qg + r$ with $\deg(r) < \deg(g)$. Assuming that $\deg(g) \neq 0$ show that

$$\mathrm{res}_x(f, g) = (-1)^{\deg(f)\deg(g)} LC(g)^{\deg(f)-\deg(r)} \mathrm{res}_x(g, r).$$

Exercise 22. Use resultants to find the implicit representation, i.e. a polynomial equation just in x, y, and z of the parametrized surface

$$\begin{array}{rcl} x & = & 1 + s + t + st \\ y & = & 2 + s + st + t^2 \\ z & = & s + t + s^2 \end{array}$$

Exercise 23. Prove the following lemma [Lemma 4.2.3].[1]
Let $a, b \in \mathbb{Z}[x_1, \ldots, x_{n-2}][y][x]^*$ and $r \in \mathbb{Z}$ such that $y - r$ does not divide both $\mathrm{lc}_x(a)$ and $\mathrm{lc}_x(b)$. Let $c = \gcd(a, b)$.

1. $\deg_x(\gcd(a_{y-r}, b_{y-r})) \geq \deg_x(\gcd(a, b))$.
2. If $y - r \not| \, \mathrm{res}_x(\frac{a}{c}, \frac{b}{c})$ then $\gcd(a_{y-r}, b_{y-r}) = c_{y-r}$.

Exercise 24. Apply the modular algorithm for computing the resultant $\mathrm{res}_y(a, b)$ of the polynomials

$$\begin{array}{rcl} a(x, y) & = & xy^2 - x^3y - 2x^2y + xy + 2x^4 - 2x^2 \\ b(x, y) & = & 2x^2y^2 - 4x^3y + 4x^4. \end{array}$$

Exercise 25. According to Theorem 4.3.3 a solution of $\mathrm{res}_{x_r}(a, b) = 0$ can be extended to a common zero of a and b, in case the leading coefficient of a or b is a nonzero constant. This can be achieved by a suitable change of coordinates. Work out an algorithm for solving systems of algebraic equations by resultants along these lines.

[1] F. Winkler, Polynomial Algorithms in Computer Algebra