to be prepared for 11.10.2016

Exercise 1. Consider the integers $a = 215712,\ b = 739914$. Determine the gcd of a and b and integers s, t such that $\gcd(a, b) = sa + tb$.

Exercise 2. Consider the two polynomials over $\mathbb{Z}$

$$\begin{aligned} f(x) &= 6x^5 + 2x^4 - 19x^3 - 6x^2 + 15x + 9 \\ g(x) &= 5x^4 - 4x^3 + 2x^2 - 2x - 2\ . \end{aligned}$$

Compute $\gcd(f(x), g(x))$.

Exercise 3. Compute the gcd of the bivariate integer polynomials

$$\begin{aligned} f(x, y) &= y^6 + xy^5 + x^3y - xy + x^4 - x^2, \\ g(x, y) &= xy^5 - 2y^5 + x^2y^4 - 2xy^4 + xy^2 + x^2y \end{aligned}$$

Exercise 4. Explain why the following is or is not a Euclidean domain.

1. $\mathbb{Z}$ with degree function $\delta(n) = |n|$;
2. $\mathbb{Q}$ with $\delta(r) = |r|$;
3. $\mathbb{Z}[i]$ with $\delta(z) = |z|^2$;
4. $k[x]$ where k is a field, with $\delta(f) = \deg(f)$;
5. $k[x, y]$ where k is a field, with $\delta(f) = \deg(f)$ (the total degree of f).

Exercise 5. Consider the ring $\mathbb{Z}$ of integers and an ideal I generated by finitely many numbers $a_1, \ldots, a_n$. As $\mathbb{Z}$ is a principal ideal domain there must be a single generator b for I.

1. Describe a procedure for finding b when arbitrary generators $a_1, \ldots, a_n$ of I are given as an input.
2. Compute a single generator for the ideal $I = \langle 33600, 784080, 214500 \rangle$.