

to be prepared for 10.01.2017

Exercise 42. Compute the normed reduced Gröbner basis for the ideal

$$I = \langle xz - 3x^2 + x + 6x^3 + 1, x^2 + y^2 - 2, x^5 - 6x^3 + x^2 - 1 \rangle$$

in $\mathbb{Q}[x, y, z]$ w.r.t. the lexicographic ordering with $x < y < z$. What is $\dim \mathbb{Q}[x, y, z]/I$?

Exercise 43. Let I be the ideal in $\mathbb{Q}[x, y, z]$ generated by

$$\begin{aligned} f_1 &= x^4 - 2x + z + 1 \\ f_2 &= x^2 + y^2 - 2 \\ f_3 &= x^5 - 6x^3 + x^2 - 1 \end{aligned}$$

1. How many solutions does the system $f_1 = f_2 = f_3 = 0$ have?
2. Give a basis for the vector space $\mathbb{Q}[x, y, z]/I$ over \mathbb{Q} .

Exercise 44. Consider the following system of algebraic equations

$$\begin{aligned} f_1 &= x^2 + y^2 + z^2 - t^2 \\ f_2 &= x^2 - y^2 + z^2 - t^2 \\ f_3 &= y^3 - 6y^2 + 12y - 6 \end{aligned}$$

Decide whether it has a solution.

Exercise 45. Given are the polynomials in $\mathbb{Q}[x, y, z, t]$

$$\begin{aligned} f_1 &= xyz t + x^2 y - z \\ f_2 &= x^3 y - xy + z^4 t \\ f_3 &= x^2 + y^2 - z^2 - t^2 \end{aligned}$$

Compute a Gröbner basis of the ideal $\langle f_1, f_2, f_3 \rangle \cap \mathbb{Q}[z, t]$.

Exercise 46. Consider the ideal $I \subseteq \mathbb{R}[x, y]$ generated by the polynomials

$$x^2 + y^2 - 1, \quad x^2 - y^2 - 1.$$

Decide whether I coincides with its radical.