to be prepared for 08.11.2016

Exercise 16. Let R be a unique factorization domain, $r \in R$ and x, y indeterminates. Consider the ring $S = R[y]/\langle y - r\rangle[x]$. Show that any two elements of S have a greatest common divisor in S.

Exercise 17. Consider the bivariate polynomials

$$\begin{aligned} f(x,y) &= x^2y^3 - 5xy^3 + 6y^3 - 6xy^2 + 18y^2 + 2xy - 4y - 12, \\ g(x,y) &= x^2y^3 + 3xy^3 - 10y^3 - 6xy^2 - 30y^2 - 4xy + 8y + 24. \end{aligned}$$

Compute the gcd of f and g by the modular algorithm. Take care of leading coefficients.

Exercise 18. Consider polyomials $f, g \in k[x]$ of positive degrees m, n respectively. Let I denote the ideal in $k[x]$ generated by f, and let μ denote the multiplication map

$$\mu\colon k[x]/I \longrightarrow k[x]/I, \quad h + I \mapsto gh + I.$$

Demonstrate that $\mathrm{res}_x(f, g) = LC(f)^{\deg(g)} \det(\mu)$.

Exercise 19. Let $f, g \in k[x]$ be polynomials whose roots are $\zeta_1, \ldots, \zeta_m$ and $\eta_1, \ldots, \eta_n$ respectively. Prove that

$$\mathrm{res}_x(f, g) = LC(f)^{\deg(g)} LC(g)^{\deg(f)} \prod_{i=1}^{m}\prod_{j=1}^{n}(\zeta_i - \eta_j)$$

Exercise 20. $f, g \in k[x]$, divide f by g, $f = qg + r$ with $\deg(r) < \deg(g)$. Assuming that $\deg(g) \neq 0$ show that

$$\mathrm{res}_x(f, g) = (-1)^{\deg(f)\deg(g)} LC(g)^{\deg(f) - \deg(r)} \mathrm{res}_x(g, r).$$