TSW
Exam Proof Training 19 Dec 2008

Name:

Note: the proofs should be done in natural style by using the techniques discussed in the lecture. Note also that we use $P[x]$ and $f[x]$ for "P applied to x" and "f applied to x", respectively (instead of the usual $P(x), f(x)$).

1. Prove: $((P \Rightarrow R) \wedge (Q \Rightarrow R)) \Leftrightarrow ((P \vee Q) \Rightarrow R)$

2. Prove: $((\exists_x P[x]) \Rightarrow Q) \Rightarrow \forall_x (P[x] \Rightarrow Q)$

3. For each symbol occuring in the following formula, specify whether it is a: logical quantifier, logical connective, predicate symbol, function symbol, variable, or constant. (Note that functions and predicates can also be constant or variable.)

$$\forall_f D[f] \;\Leftrightarrow \forall_{\epsilon>0}\ \exists_{\delta>0}\ \forall_x\ (0 < |x| < \delta\ \Rightarrow\ |\frac{f[x]}{x}| < \epsilon)$$

4. In the previous definition, D denotes differentiability (in zero), f denotes a real function of real argument, and x, δ and ϵ denote real numbers. Formulate and prove the statement that the sum of differentiable functions in differentiable. Emphasize the properties from the theory of real numbers which are necessary for this proof (definition of $f_1 + f_2$, properties of minimum and of absolute value, etc.).

5. Formulate the notions of binary relation over a set, reflexivity, symmetry, transitivity, and equivalence.

6. In the following formulae, n and s stand for natural numbers:

$$F[0] = 0$$
$$\forall_{n>0}\ F[n] = n + F[n-1]$$
$$\forall_s\ G[s,\ 0] = s$$
$$\forall_{n>0}\ \forall_s\ G[s,\ n] = G[s+n,\ n-1]$$

Use these equalities as rewrite rules in order to compute the expressions: $F[3]$ and $G[0,\ 3]$.

7. Using the formulae above, prove $\forall_n\ F[n] = G[0,\ n]$.
Hint: prove first $\forall_n\ \forall_s\ G[s,\ n] = s + F[n]$. For proving the latter, consider the predicate $P[n]$ defined as $\forall_s\ G[s,\ n] = s + F[n]$ and use the induction principle for natural numbers in order to prove $\forall_n\ P[n]$. (One must prove $P[0]$ and $\forall_n\ (P[n] \Rightarrow P[n+1])$.) Note that for proving equalities it is enough to transform both sides by using known equalities as rewrite rules (and, of course, the appropriate properties of natural numbers).