to be prepared for 10.12.2013

Exercise 40. Use resultants to find the implicit representation, i.e. a polynomial equation just in x, y, and z of the parametrized surface

$$\begin{aligned} x &= 1 + s + t + st \\ y &= 2 + s + st + t^2 \\ z &= s + t + s^2 \end{aligned}$$

Exercise 41. Let I be a unique factorization domain, $f, g \in I[x]$ with $\deg(f) > 0, \deg(g) > 0$. Show that there are polynomials $a, b \in I[x] \backslash 0$ such that $\mathrm{res}(f, g) = af + bg$.

Exercise 42. Consider polyomials $f, g \in k[x]$ of positive degrees m, n respectively. Let I denote the ideal in $k[x]$ generated by f, and let μ denote the multiplication map

$$\mu\colon k[x]/I \longrightarrow k[x]/I, \quad h + I \mapsto gh + I.$$

Demonstrate that $\mathrm{res}_x(f, g) = LC(f)^{\deg(g)} \det(\mu)$.

Exercise 43. Let $f, g \in k[x]$ be polynomials whose roots are $\zeta_1, \ldots, \zeta_m$ and $\eta_1, \ldots, \eta_n$ respectively. Prove that

$$\mathrm{res}_x(f, g) = LC(f)^{\deg(g)} LC(g)^{\deg(f)} \prod_{i=1}^{m} \prod_{j=1}^{n} (\zeta_i - \eta_j)$$

Exercise 44. $f, g \in k[x]$, divide f by g, $f = qg + r$ with $\deg(r) < \deg(g)$. Assuming that $\deg(g) \neq 0$ show that

$$\mathrm{res}_x(f, g) = (-1)^{\deg(f)\deg(g)} LC(g)^{\deg(f) - \deg(r)} \mathrm{res}_x(g, r).$$