Automated Reasoning Exercises
(326.094, WS 2013)

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Problem 1: Let $S := \{p(a, b), \neg p(b, a), a \approx b\}$. Find all $\mathcal{E}$-interpretations of S. Is S $\mathcal{E}$-unsatisfiable? Why?

Problem 2: Let $S := \{p(x) \vee x \approx b, \neg p(a), p(b)\}$. Find an $\mathcal{E}$-unsatisfiable set S' of ground instances of clauses in S.

Problem 3: Let $C_1 := p(f(x, g(x))) \vee q(x)$ and $C_2 := a \approx b \vee g(a) \approx a \vee f(a, g(a)) \approx b$. Find all binary paramodulants of C_1 and C_2.

Problem 4: Consider the following set S of clauses:

(1) $r(a) \vee r(b)$

(2) $\neg p(y) \vee s(a, y)$

(3) $\neg r(x) \vee \neg q(y) \vee \neg s(x, y)$

(4) $p(a) \vee \neg q(a)$

(5) $q(b) \vee \neg r(b)$

(6) $a \approx b$

Using the $\mathcal{PR}$ inference system discussed in the class, find a refutation from S. Can you find an input refutation by $\mathcal{PR}$?

Problem 5: Consider the following set S of clauses:

(1) $p(b) \vee q(a)$

(2) $p(a) \vee \neg q(b)$

(3) $\neg p(a) \vee q(b)$

(4) $\neg p(b) \vee \neg q(a)$

(5) $a \approx b$

Find a linear refutation from S with the top clause (5). Repeat the same with the top clause (1).

Problem 6: Is the TRS consisting of the rewrite rules

$$d(0) \to 0$$
$$d(s(x)) \to s(s(d(x)))$$

terminating?

Problem 7: Use LPO to show termination of the following TRS:

$$\{f(g(g(x)), y) \to f(g(x), f(x, y)), \quad f(g(x), g(y)) \to f(f(x, x), f(y, y))\}$$

Problem 8: Prove $i(i(x)) \approx f(i(a), f(b, a))$ from the equalities

$$f(x, e) \approx x$$
$$f(x, i(x)) \approx e$$
$$f(x, f(y, z)) \approx f(f(x, y), z)$$
$$f(x, y) \approx f(y, x)$$

using the ordered completion procedure, where the reduction order is LPO
with the precedence $i > f > e$. Show every step of the derivation, indicating
the applied transformation rule.