Exercises discussed on January 29, 2013

All examples are meant to be carried out with the help of a computer algebra system!

49. Compare the output of the three commands described below of the packages zb.m and HolonomicFunctions.m for the sum

$$s(n) = \sum_{k=0}^{n} \binom{n}{2k}\binom{2k}{k} 4^{-k}.$$

Can you determine a closed form solution for $s(n)$?

50. Zeilberger's algorithm does not always return the minimal order recurrence. Let

$$s_d(n) = \sum_{k=0}^{n} (-1)^k \binom{n}{k}\binom{dk}{n}.$$

   (a) Use the Annihilator command to derive a recurrence satisfied by $s_2(n)$.
   (b) Use guessing to find a shorter recurrence for $s_2(n)$.
   (c) Use closure properties to show that the guessed recurrence is correct.
   (d) Find a closed form solution for $s_2(n)$.
   (e) Try HolonomicFunctions and zb for the cases $d = 3, 4, 5, \ldots$. What are your observations?

- Examples on how to use Zeilberger's algorithm with the package zb.m:

In[1]:= **Zb[Binomial[n, k], {k, 0, n}, n]**
If 'n' is a natural number, then:

Out[1]= $\{2\mathrm{SUM}[n] - \mathrm{SUM}[1+n] == 0\}$

In[2]:= **Zb[Binomial[n, k], {k, 0, Infinity}, n]**

Out[2]= $\{2\mathrm{SUM}[n] - \mathrm{SUM}[1+n] == 0\}$

The latter works because we have natural boundaries in this case.

- Examples on how to use HolonomicFunctions: Plug in the sum into the Annihilator command:

In[3]:= **Annihilator[Sum[Binomial[n, k], {k, 0, n}], {S[n]}]**

Out[3]= $\{S_n - 2\}$

Given the summand $a(n,k)$, the command CreativeTelescoping returns two operators P, Q such that

$$P \bullet a(n,k) - \Delta_k \left(Q \bullet a(n,k)\right) = 0.$$

In[4]:= **CreativeTelescoping[Binomial[n, k], S[k] − 1, S[n]]**

Out[4]= $\{\{S_n - 2\}, \{-\frac{k}{k-n-1}\}\}$

The argument $S[k] - 1$ indicates that summation is carried out w.r.t. k, and $S[n]$ that a recurrence in n is to be determined.