Exercises discussed on January 8, 2013

39. Determine all polynomial solutions of the recurrence

$$(4n+9)a(n) - 4(n+1)a(n+1) + 3a(n+2) = 0, \qquad a(0) = -1,\ a(1) = 0.$$

40. Implement a program in your favourite computer algebra system that sums a given polynomial sequence using

    (a) falling factorial representation.

    (b) interpolation (you may use built-in commands to execute the interpolation, e.g., in Mathematica the command InterpolatingPolynomial).

    Compute some test cases, in particular compare the timings for the sparse and dense polynomial given in testcases.m.

41. Express $s_n = \sum_{k=0}^{n} a_k$ in terms of $a_n, a_{n+1}, \ldots$, where the sequence $\big(a_n\big)_{n\geq 0}$ is given by the recurrence

$$a_{n+4} - a_{n+3} - 3a_{n+2} + 5a_{n+1} - 2a_n = 0, \qquad a_0 = 3, a_1 = -4, a_2 = 9, a_3 - 12.$$

42. Express $s_n = \sum_{k=0}^{n} a_k$ in terms of $a_n, a_{n+1}, \ldots$, where the sequence $\big(a_n\big)_{n\geq 0}$ is given by the recurrence

$$a_{n+2} + a_{n+1} - 6a_n = 0, \qquad a_0 = 1, a_1 = -1.$$

Enjoy the vacations and Happy New Year!