Exercises discussed on November 27, 2012

31. Determine the asymptotics of

$$\frac{3^n}{4n+1}\binom{3n}{n+1}^2\binom{6n}{2n}^{-1}.$$

32. Show that if $\big(a_n\big)_{n\geq 0}$ is holonomic, then $s_n = \sum_{k=0}^{n} a_k$ is holonomic.

33. Use Mallinger's package GeneratingFunctions to

    (a) compute the defining differential equation for $y(x) = \frac{x}{\sqrt{1-4x}}$ starting from the algebraic equation

$$(1-4x)y(x)^2 - x^2 = 0;$$

    (b) derive a recurrence relation for the coefficients a_n of $y(x) = \sum_{n\geq 0} a_n x^n$ starting from the differential equation computed in (a).

    Solve the recurrence computed in (b) using your favourite computer algebra system.

34. Prove Euler's transform: For $y \in \mathbb{C}[\![x]\!]$:

$$\langle x^n\rangle \frac{1}{1-x} y\left(\frac{x}{x-1}\right) = \sum_{k=0}^{n}\binom{n}{k}(-1)^k\langle x^k\rangle y(x).$$