Exercises discussed on October 16, 2012

6. Let
$$g(n) = \sum_{k=0}^{n-1} \frac{2k}{(k+1)(k+2)}.$$
Show that
$$g(n) = 2H_n + \frac{4}{n+1} - 4$$
   (a) by hand.
   (b) using a computer algebra system.

7. Show that $H_n \sim \log n$ as n tends to infinity without using Euler's result.

(BP1) Show that the harmonic numbers cannot be expressed as a rational function, i.e., show that $H_n \notin \mathbb{C}(n)$.

8. Determine the generating function $F(x) = \sum_{n\geq 0} c_n x^n$ for
   (a) $c_n = n^2$.
   (b) $c_n = \frac{1}{n+1}$.

9. Show that $\left(\mathbb{K}^{\mathbb{N}}, +, \cdot\right)$ is a commutative ring with one.

10. Show that $(\mathbb{K}[\![x]\!], +, \cdot)$ is an integral domain.

(BP) indicates *Bonus problem*, i.e., it brings extra credits.