Exercises discussed on January 31, 2012

58. Let $a(i,j)$ denote the number of paths starting at $(0,0)$ and ending at (i,j) that a rook may take who is only moving up or to the right (as in the lecture). Show that $a(1,j) = 2^{j-2}(j+3)$.

59. Implement the three recurrences presented in the lecture to compute $a(i,j)$ and compare their efficiency.

60. Let $b(i,j)$ be the number of rook walks starting at $(0,0)$ and ending at (i,j), where the rook is allowed to move only one step at a time in directions either up, right or diagonally right up (i.e., $(1,0), (0,1), (1,1)$). Determine a recurrence relation satisfied by $b(i,j)$ with sufficiently many initial values.

61. Let
$$f(x,y) = \frac{(x-1)(x-y)}{x\,(2x^2 - 3xy - x + 2y)}.$$
Use CreativeTelescoping (HolonomicFunctions.m) to derive a differential equation in y of $F(y) = \int_D f(x,y)\mathrm{dx}$, where you may assume that the domain has natural boundaries and you can safely ignore the right hand side. Use holonomic closure properties to find a recurrence for $\beta(n)$, where $F(y) = \sum_{n\geq 0} \beta(n) y^n$.