Exercises discussed on December 13, 2011

41. Let $a(x) = \sum_{n\geq 0} a_n x^n \in \mathbb{K}[\![x]\!]$. Show that if $\left(a_n\right)_{n\geq 0}$ is holonomic of order r and degree d, then $a(x)$ is holonomic of order at most d and degree at most $r+d$.

42. Show that if $\left(a_n\right)_{n\geq 0}$ is holonomic, then $s_n = \sum_{k=0}^{n} a_k$ is holonomic.

43. Use Mallinger's package GeneratingFunctions to

    (a) compute the defining differential equation for $y(x) = \frac{x}{\sqrt{1-4x}}$ starting from the algebraic equation

    $$(1-4x)y(x)^2 - x^2 = 0;$$

    (b) derive a recurrence relation for the coefficients a_n of $y(x) = \sum_{n\geq 0} a_n x^n$ starting from the differential equation computed in (a).

    Solve the recurrence computed in (b) using your favourite computer algebra system.

44. Prove Euler's transform: For $y \in \mathbb{C}[\![x]\!]$:

$$\langle x^n \rangle \frac{1}{1-x} y\left(\frac{x}{x-1}\right) = \sum_{k=0}^{n} \binom{n}{k} (-1)^k \langle x^k \rangle y(x).$$