Exercises discussed on December 6, 2011

36. Can $f(x) = \sin(x) + \cos(x)$ be expressed as a hypergeometric series?

37. Show that a sequence $\big(a_n\big)_{n\geq 0}$ is holonomic if and only if there exist polynomials $p_0, \ldots, p_r \in \mathbb{K}[x]$ and $q \in \mathbb{K}[x]$ such that

$$p_r(n)a_{n+r} + \cdots + p_1(n)a_{n+1} + p_0(n)a_n = q(n), \qquad n \in \mathbb{N}.$$

38. Implement a procedure in your favourite computer algebra system that given polynomials $a_0(n), a_1(n), a_2(n), c(n)$ returns the degree bound D of Algorithm POLY, i.e., the degree bound for potential polynomial solutions $y(n)$ of

$$a_0(n)y(n) + a_1(n)y(n+1) + a_2(n)y(n+2) = c(n).$$

39. Determine all polynomial solutions of the recurrence

$$(n+1)a(n) - (2n+3)a(n+1) + (n+2)a(n+2) = 0.$$

40. Determine all polynomial solutions of the recurrence

$$(4n+9)a(n) - 4(n+1)a(n+1) + 3a(n+2) = 0, \qquad a(0) = -1, \quad a(1) = 0$$